Park Academy West London
an Aspirations Academy

Maths

A level Mathematics is often thought of as a subject of complicated calculations. However, calculations form only a small part of this rigorous discipline which requires clear thinking and the development of specific ideas into generalised solutions.

On one hand A level Mathematics deals with highly abstract topics which require considerable imagination combined with the discipline of 'proof'. On the other hand mathematics underpins virtually all the practical developments in science, IT and economics which have formed our modern world.

A level Mathematics gives you the opportunity to study topics such as geometry, calculus and trigonometry (pure mathematics) and to use these ideas within the 'applied' topics such as mechanics and statistics.

Task: Indices and surds

You can apply the rules of indices and surds to simplify algebraic expressions. The following expressions can be simplified in **index form:**

Key point

$x^a \times x^b = x^{a+b}$ $\quad x^a \div x^b = x^{a-b}$ $\quad (x^a)^b = x^{ab}$

Example 1

Simplify these expressions. **a** $2x^3 \times 3x^5$ **b** $12x^7 \div 4x^3$ **c** $(3x^5)^3$

a $2x^3 \times 3x^5 = 6x^{3+5}$

$= 6x^8$

Multiply the coefficients together and use $x^a \times x^b = x^{a+b}$

b $12x^7 \div 4x^6 = \dfrac{12x^7}{4x^6}$

$= 3x$

Since $\frac{12}{4} = 3$ and $x^a \div x^b = x^{a-b}$ so $\frac{x^7}{x^6} = x^1$ which we just write as x

c $(3x^5)^3 = 3^3(x^5)^3$

Both the 3 and the x^5 must be raised to the power 3

$= 27x^{15}$

Since $(x^a)^b = x^{ab}$

Simplify these expressions:

a. $5x^3 \times 2x^7$ b. $18x^9 \div 3x^2$ c. $(2x^6)^4$ d. $\left(\frac{x^3}{3}\right)^2$

Roots can also be expressed using indices, such as the square root of x is written as $\sqrt{x} = x^{\frac{1}{2}}$

In general:

Key point

The nth root of x is written $\sqrt[n]{x} = x^{\frac{1}{n}}$, and this can be raised to a power to give $\sqrt[n]{x^m} = x^{\frac{m}{n}}$

Key point

A power of -1 indicates a reciprocal, so $x^{-1} = \frac{1}{x}$ and, in general, $x^{-n} = \frac{1}{x^n}$

Example 2

Evaluate each of these without using a calculator.

a $25^{0.5}$ **b** 6^{-2} **c** $8^{\frac{2}{3}}$

a $25^{0.5} = 25^{\frac{1}{2}}$

$= \sqrt{25}$

$= 5$

Since a power of $\frac{1}{2}$ represents a square root.

b $6^{-2} = (6^2)^{-1}$

$= \frac{1}{6^2}$

Since a power of -1 represents a reciprocal.

$= \frac{1}{36}$

c $8^{\frac{2}{3}} = \left(8^{\frac{1}{3}}\right)^2$

Always calculate a root before a power.

$= 2^2$

Since the cube root of 8 is 2

$= 4$

Evaluate each of these without a calculator:

b. $36^{\frac{1}{2}}$ b. $27^{\frac{2}{3}}$ c. $64^{-0.5}$ d. $\left(\frac{1}{2}\right)^4$

Example 3

Write these expressions in simplified index form.

a $\sqrt[3]{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{2x}{\sqrt{x}}$

a $\sqrt[3]{x} = x^{\frac{1}{3}}$

b $\frac{2}{x^3} = 2x^{-3}$

c $\frac{2x}{\sqrt{x}} = \frac{2x}{x^{\frac{1}{2}}}$ Since $\sqrt{x} = x^{\frac{1}{2}}$

$= 2x^{1-\frac{1}{2}}$ Subtract the powers, remembering that $x = x^1$

$= 2x^{\frac{1}{2}}$

Write these expressions in simplified index form:

a. $\sqrt[5]{x^2}$ b. $\frac{3}{\sqrt{x}}$ c. $\frac{3x^2}{\sqrt{x}}$ d. $\frac{\sqrt{x}}{3x}$

A **surd** is an irrational number involving a root, for example $\sqrt{2}$ or $\sqrt[3]{7}$.

You can multiply and divided using the rules:

Key point

$$\sqrt{a}\times\sqrt{b}=\sqrt{ab} \text{ and } \frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

You can simplify surds by finding square-number factors, for example $\sqrt{12} = \sqrt{4}\,\sqrt{3} = 2\sqrt{3}$. It may also be possible to simplify expressions involving surds by collecting like terms or by **rationalising the denominator**. Rationalising the denominator means rearranging the expression to remove any roots from the denominator.

Key point

To rationalise the denominator, multiply both the numerator and denominator by a suitable expression:

$\frac{1}{\sqrt{a}}\times\frac{\sqrt{a}}{\sqrt{a}}=\frac{\sqrt{a}}{a}$, (multiply numerator and denominator by $\sqrt{a}$)

$\frac{1}{a+\sqrt{b}}\times\frac{a-\sqrt{b}}{a-\sqrt{b}}=\frac{a-\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a-\sqrt{b}$)

$\frac{1}{a-\sqrt{b}}\times\frac{a+\sqrt{b}}{a+\sqrt{b}}=\frac{a+\sqrt{b}}{a^2-b}$, (multiply numerator and denominator by $a+\sqrt{b}$)

Example 4

Simplify these expressions without using a calculator.

a $\sqrt{18}+5\sqrt{2}$ **b** $\frac{6}{\sqrt{3}}$ **c** $\frac{2}{1-\sqrt{5}}$

a $\sqrt{18}=\sqrt{9}\sqrt{2}$ — 9 is a square-number factor of 18 so you can simplify $\sqrt{18}$

$=3\sqrt{2}$

Therefore $\sqrt{18}+5\sqrt{2}=3\sqrt{2}+5\sqrt{2}$

$=8\sqrt{2}$ — Collect like terms.

b $\frac{6}{\sqrt{3}}=\frac{6\sqrt{3}}{\sqrt{3}\sqrt{3}}$ — Rationalise the denominator by multiplying numerator and denominator by $\sqrt{3}$

$=\frac{6\sqrt{3}}{3}$

$=2\sqrt{3}$ — Since $6\div 3=2$

c $\frac{2}{1-\sqrt{5}}=\frac{2(1+\sqrt{5})}{(1-\sqrt{5})(1+\sqrt{5})}$ — Rationalise the denominator by multiplying numerator and denominator by $1+\sqrt{5}$

$=\frac{2(1+\sqrt{5})}{-4}$ — $(1-\sqrt{5})(1+\sqrt{5})=1-\sqrt{5}+\sqrt{5}-5$
$=1-5=-4$

$=-\frac{1}{2}(1+\sqrt{5})$

Simplify these expressions without using a calculator.

a. $3\sqrt{28} - \sqrt{7}$ b. $\frac{4}{\sqrt{3}}$ c. $\frac{3}{1 + \sqrt{2}}$ d. $\frac{\sqrt{5}}{\sqrt{5}-2}$

1. Evaluate each of these without using a calculator:

   a. $49^{\frac{1}{2}}$

   b. $27^{\frac{1}{3}}$

   c. 5^{-1}

   d. $64^{\frac{1}{3}}$

   e. $9^{\frac{3}{2}}$

   f. $16^{\frac{3}{4}}$

   g. $125^{\frac{2}{3}}$

   h. $\left(\frac{1}{2}\right)^{3}$

   i. $\left(\frac{1}{9}\right)^{-2}$

   j. $\left(\frac{4}{9}\right)^{\frac{1}{2}}$

   k. $\left(\frac{9}{16}\right)^{-0.5}$

   l. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

2. Simplify these expressions fully without using a calculator.

a. $\sqrt{8}$

b. $\sqrt{75}$

c. $2\sqrt{24}$

d. $3\sqrt{48}$

e. $\sqrt{20} + \sqrt{5}$

f. $\sqrt{27} - \sqrt{12}$

g. $5\sqrt{32} - 3\sqrt{8}$

h. $\sqrt{50} + 3\sqrt{125}$

i. $\sqrt{68} + 3\sqrt{17}$

j. $3\sqrt{72} - \sqrt{32}$

k. $4\sqrt{18} - 2\sqrt{3}$

l. $6\sqrt{5} + \sqrt{50}$

3. Simplify these expressions fully without using a calculator.

a. $\frac{1}{\sqrt{7}}$

b. $\frac{2}{\sqrt{8}}$

c. $\frac{12}{\sqrt{3}}$

d. $\frac{\sqrt{8}}{\sqrt{12}}$

e. $\frac{1}{1+\sqrt{3}}$

f. $\frac{2}{1+\sqrt{2}}$

g. $\frac{8}{1-\sqrt{5}}$

h. $\frac{2}{\sqrt{5}-1}$

i. $\frac{\sqrt{2}}{2+\sqrt{3}}$

j. $\frac{2\sqrt{3}}{\sqrt{6}-2}$

k. $\frac{1+\sqrt{2}}{1-\sqrt{2}}$

l. $\frac{3+\sqrt{5}}{\sqrt{5}-3}$

4. Expand the brackets and fully simplify each expression:

   a. $(1+\sqrt{2})(3+\sqrt{2})$

   b. $(1+\sqrt{2})(3-\sqrt{2})$

   c. $(1-\sqrt{2})(3+\sqrt{2})$

   d. $(1-\sqrt{2})(3-\sqrt{2})$

   e. $(\sqrt{3}+2)(4+\sqrt{3})$

   f. $(\sqrt{3}+2)(4-\sqrt{3})$

   g. $(\sqrt{3}-2)(4+\sqrt{3})$

   h. $(\sqrt{3}-2)(4-\sqrt{3})$

   i. $(\sqrt{6}+1)(\sqrt{2}+3)$

   j. $(\sqrt{6}+1)(\sqrt{2}-3)$

   k. $(\sqrt{6}-1)(\sqrt{2}+3)$

   l. $(\sqrt{6}-1)(\sqrt{2}-3)$

5. Write each of these expressions in simplified index form.

a. $x^3 \times x^7$

b. $7x^5 \times 3x^6$

c. $5x^4 \times 8x^7$

d. $x^8 \div x^2$

e. $8x^7 \div 2x^9$

f. $3x^8 \div 12x^7$

g. $(x^5)^7$

h. $(x^2)^{-5}$

i. $(3x^2)^4$

j. $(6x^5)^2$

k. $\sqrt{x^3}$

l. $\sqrt[4]{x^5}$

m. $\frac{5\sqrt{x}}{x}$

n. $2x\sqrt{x}$

o. $\frac{x^2}{3\sqrt{x}}$

p. $x^3(x^5 - 1)$

q. $x^3(\sqrt{x} + 2)$

r. $\frac{x+2}{x^3}$

s. $\frac{\sqrt{x}+3}{x}$

t. $\frac{(3-x^3)}{\sqrt{x}}$

u. $(\sqrt{x} + 3)^2$

v. $\frac{3+\sqrt{x}}{x^2}$

w. $\frac{1-x}{2\sqrt{x}}$

x. $\frac{\sqrt{x}+2}{3x^3}$

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Task: Solving linear equations and rearranging formulae

This topic recaps the **balance** method to solve problems involving linear equations, and both the **elimination** and **substitution** methods to solve linear simultaneous equations. You can solve linear equations and inequalities using the **balance** method where the same operation is applied to both sides.

Example 1

Solve the equation $7x-5=3x-2$

$4x-5=-2$ Subtract $3x$ from both sides of the equation.

$4x=3$ Add 5 to both sides of the equation.

$x=\frac{3}{4}$ Divide both sides of the equation by 4

Solve the equation $3x+8=5x-6$

Example 2

Solve the inequality $5(x-2)\leq 2x+1$

$5x-10\leq 2x+1$ First expand the brackets.

$3x-10\leq 1$ Subtract $2x$ from both sides.

$3x\leq 11$ Add 10 to both sides.

$x\leq\frac{11}{3}$ Divide both sides by 3

Solve the inequality $7x-4>x+8$

When solving inequalities, remember that multiplying or dividing by a negative number will reverse the inequality sign. For example, $5 > 3$ but $-5 < -3$

Equations and formulae can be rearranged using the same method as for solving equations.

Example 3

Rearrange $Ax - 3 = \frac{x+B}{2}$ to make x the subject.

$2Ax - 6 = x + B$ — Multiply both sides by 2

$2Ax - 6 - x = B$ — Subtract x from both sides.

$2Ax - x = B + 6$ — Add 6 to both sides.

$x(2A - 1) = B + 6$ — Factorise the side involving x

$x = \frac{B+6}{2A-1}$ — Divide both sides by $(2A - 1)$ to make x the subject.

Rearrange $3(x + A) = Bx + 1$ to make x the subject.

You can solve linear simultaneous equations using the **elimination** method, as shown in Example 4. The solutions to simultaneous equations give the point of intersection between the lines represented by the two equations.

Example 4

Solve the simultaneous equations $5x - 4y = 17$, $3x + 8y = 5$

$15x + 40y = 25 \quad (1)$ — Multiply the second equation by 5

$15x - 12y = 51 \quad (2)$ — Multiply the first equation by 3

$(1) - (2): 52y = -26$ — Subtract equation (2) from equation (1) to eliminate x

$y = -\frac{1}{2}$

$5x - 4\left(-\frac{1}{2}\right) = 17$ — Substitute $y = -\frac{1}{2}$ into one of the original equations.

$5x + 2 = 17$ — Solve this equation to find the value of x

$5x = 15$

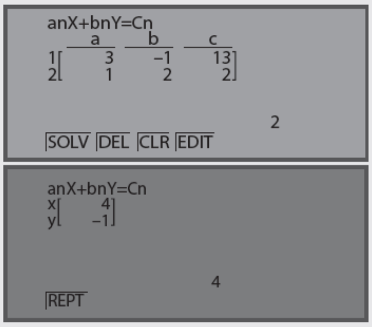
$x = 3$

Solve the simultaneous equations $2x + 5y = 1,\ 3x - 2y = -27$

Calculator

Try it on your calculator

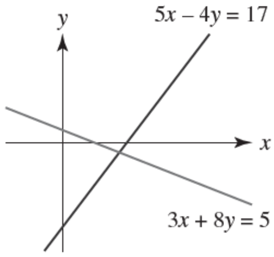
You can use a calculator to solve linear simultaneous equations.



Activity

Find out how to solve the simultaneous equations $3x - y = 13$ and $x + 2y = 2$ on *your* calculator.

The example shows you that the lines $5x - 4y = 17$ and $3x + 8y = 5$ intersect at the point $\left(3, -\frac{1}{2}\right)$



If you are given the equation of two lines where y is the subject then the easiest way to solve these simultaneously is to use the **substitution** method as shown in the next example.

Example 5

Find the point of intersection between the lines with equations $y = 2x + 5$ and $y = 7 - 3x$

$2x + 5 = 7 - 3x$ — Substitute $2x + 5$ for y in the second equation.

$5x + 5 = 7$ — Solve to find the value of x

$5x = 2$

$x = 0.4$

$y = 2(0.4) + 5$ — Substitute $x = 0.4$ into either of the original equations to find the y-coordinate.

$= 5.8$

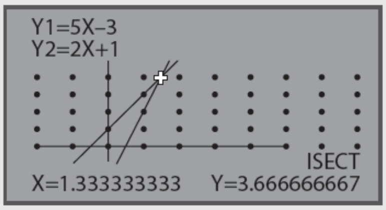
So the lines intersect at the point $(0.4, 5.8)$

Find the point of intersection between the lines $y = 3x + 4$ and $y = 6x - 2$

<u>Calculator</u>

Try it on your calculator

You can use a graphics calculator to find the point of intersection of two lines.



Activity

Find the point of intersection of the lines $y = 5x - 3$ and $y = 2x + 1$ on your graphics calculator.

1. Solve each of these linear equations.

   a. $3(2x + 9) = 7$

   b. $7 - 3x = 12$

c. $\frac{x+4}{5} = 7$

d. $2x + 7 = 5x - 6$

e. $8x - 3 = 2(3x + 1)$

f. $\frac{2x+9}{12} = x - 1$

g. $2(3x - 7) = 4x$

h. $7 - 2x = 3(4 - 5x)$

2. Solve each of these linear equalities.

a. $\frac{x}{2} + 7 \geq 5$

b. $3 - 4x < 15$

c. $5(x - 1) > 12 + x$

d. $\frac{x+1}{3} > 2$

e. $8x - 1 \leq 2x - 5$

f. $3(x + 1) \geq \frac{x-3}{2}$

g. $3(2x - 5) < 1 - x$

h. $x - (3 + 2x) \geq 2(x + 1)$

3. Rearrange each of these formulae to make x the subject.

   a. $2x + 5 = 3A - 1$

   b. $x + u = vx + 3$

   c. $\frac{3x-1}{k} = 2x$

   d. $5(x - 3m) = 2nx - 4$

e. $(1-3x)^2 = t$

f. $\frac{1}{x} = \frac{1}{p} + \frac{1}{q}$

g. $\frac{1}{x^2+k} - 6 = 4$

h. $\sqrt{x+A} = 2B$

4. Use algebra to solve each of these pairs of simultaneous equations.

   a. $5x + 12y = -6, \;\; x + 5y = 4$

   b. $7x + 5y = 14, \;\; 3x + 4y = 19$

   c. $2x - 5y = 4, \;\; 3x - 8y = 5$

d. $3x - 2y = 2,\ 8x + 3y = 4.5$

e. $5x - 2y = 11, \quad -2x + 3y = 22$

f. $8x + 5y = -0.5, \quad -6x + 4y = -3.5$

5. Use algebra to find the point of intersection between each pair of lines.

a. $y = 8 - 3x,\ \ y = 2 - 5x$

b. $y = 7x - 4,\ \ y = 3x - 2$

c. $y = 2x + 3,\ \ y = 5 - x$

d. $y + 5 = 3x, \;\; y = -5x + 7$

e. $y = \frac{1}{2}x + 3, \;\; y = 5 - 2x$

f. $y = 3(x + 2), \;\; y = 7 - 2x$