

## Y10 Higher Algebra

1 Factorise:

**a**  $12a + 8b$

**b**  $xy - 2x$

**c**  $2k^2 + 6k$

**d**  $3pq^2 - 12p^2r$

2 **a** Write down all of the whole number values of  $x$ , such that  $-5 < x \leq 3$

**b** Represent the inequality  $-5 < x \leq 3$  on a number line.

3 Remove the brackets and then simplify:

**a**  $3(5x + y) + 2(3y - 2x)$

**b**  $5(2m + 3) - 3(4 - m)$

4 Here are the first five numbers of a sequence.

3, 9, 15, 21, 27

**a** Write down the next two numbers in the sequence.

**b** Write down, in words, the term-to-term rule to continue this sequence.

**c** Write down an expression for the  $n$ th term of this sequence.

**d** What will the 20th term of the sequence be?

5 Solve the inequality  $6y - 4 \leq 2y + 7$

6 Solve the following equations:

**a**  $4(a + 3) = 6(a - 1)$

**b**  $\frac{x+1}{2} - \frac{2x-3}{5} = 1$

7 Starting with  $x = 4$ , use a trial and improvement method to find, correct to one decimal place, a solution to the equation  $x^3 + x = 84$

Show all your working.

8 **a** Copy and complete the table for  $y = x^2 - 2x - 2$

$x$	-2	-1	0	1	2	3	4
$y$	6		-2			1	

**b** Draw  $x$ - and  $y$ -axes with the  $x$ -axis from  $-2$  to  $4$  and the  $y$ -axis from  $-4$  to  $6$ .

On the axes, draw the graph of  $y = x^2 - 2x - 2$  for values of  $x$  from  $-2$  to  $4$ .

**c** Write down the equation of the line of symmetry of the graph.

**d** Write down the coordinates of the minimum point on the graph.

**e** Use your graph find the values of  $x$  when  $y = 0$

9 In each of the following, make  $a$  the subject of the formula.

**a**  $2q = p + at$

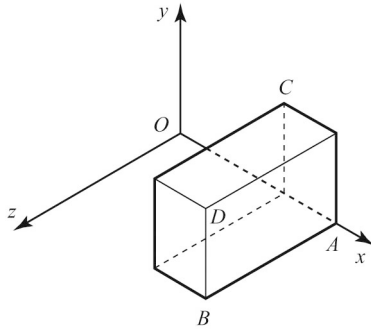
**b**  $v = \frac{3a}{2} - 5$

**c**  $d = \sqrt{3a + c}$

**d**  $4a - m = n + aq$

- 10  $M$  is the point  $(-2, 4)$  and  $N$  is the point  $(6, -8)$ .
- Find the coordinates of the midpoint of the line  $MN$ .
  - Find the gradient of the line  $MN$ .
  - Find the equation of this line.
  - Another line  $PQ$  is parallel to  $MN$  and passes through the point  $(1, 5)$ . Find the equation of  $PQ$ .

- 11 The diagram shows a cuboid.
- The coordinates of point  $A$  are  $(6, 0, 0)$ .
- The coordinates of point  $B$  are  $(6, 0, 5)$ .
- The coordinates of point  $C$  are  $(4, 3, 0)$ .
- Write down the coordinates of point  $D$ .



- 12 Solve these pairs of simultaneous equations:

a  $3x + 2y = 12$   
 $x + 2y = 2$

b  $5x - 2y = 7$   
 $x + 2y = 11$

c  $7x - 3y = 48$   
 $2x + y = 10$

d  $3x - 4y = 14$   
 $5x + 3y = -54$

- 13 a Copy and complete the table of values for  $y = x^3 - 2x^2 - 4x$

$x$	-3	-2	-1	0	1	2	3	4
$y$	-33			0			-3	

- Draw the graph of  $y = x^3 - 2x^2 - 4x$  for values of  $x$  from  $-3$  to  $4$ .
  - Use your graph to solve the equations
    - $x^3 - 2x^2 - 4x = 1$
    - $x^3 - 2x^2 - 4x = -5$
- 14 a Multiply out and simplify:
- $(x + 4)(x + 7)$
  - $(x - 6)(x + 3)$
  - $(x + 5)(x - 5)$
  - $(3x + 2)(5x - 4)$
- b Factorise:
- $x^2 + 3x - 18$
  - $x^2 - 9x + 20$
  - $2x^2 - 5x - 3$
  - $6x^2 - 27x + 30$

- 15 Simplify:

a  $\frac{x^2 - 9}{3x - 9}$

b  $\frac{2x^2 + 5x - 3}{8x - 4}$

c  $\frac{x^2 - 6x + 8}{x^2 - 4}$

16 Solve each of these quadratic equations:

**a**  $x^2 + 2x - 24 = 0$

**b**  $x^2 + 6x = 0$

**c**  $x^2 - 64 = 0$

**d**  $2x^2 + 5x - 12 = 0$

17 Solve these equations, giving each answer correct to two decimal places.

**a**  $x^2 + 7x + 8 = 0$

**b**  $x^2 - 3x + 1 = 0$

**c**  $2x^2 + 10x - 3 = 0$

18 **a** Sketch the graph of  $y = \cos x$  for values of  $x$  from  $0^\circ$  to  $360^\circ$ .

**b** Use your sketch, together with your calculator, to solve the equation  $\cos x = -0.4$ . Find all the solutions for  $x$  that lie between  $0^\circ$  and  $360^\circ$ . Give your solutions correct to 1 decimal place.

19 Solve these quadratic equations by completing the square.

**a**  $x^2 - 8x + 1 = 0$

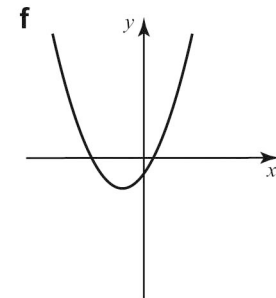
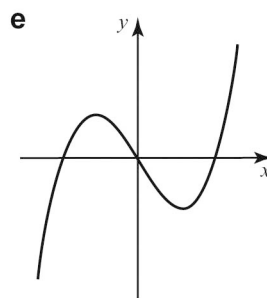
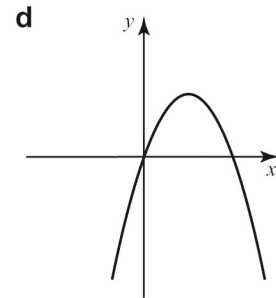
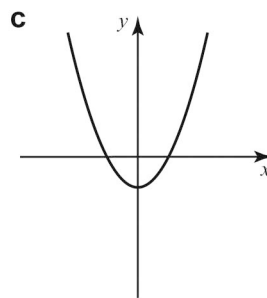
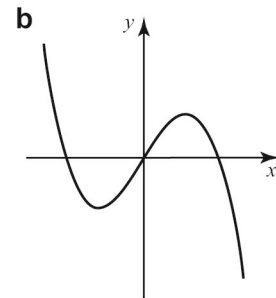
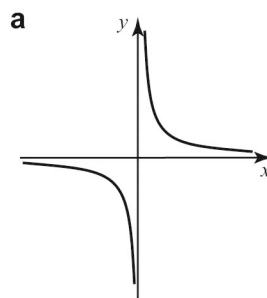
**b**  $x^2 + 4x + 2 = 0$

**c**  $x^2 + 2x - 4 = 0$

20 Each of the equations in the table represents one of the graphs A to F.

Copy and complete the table, writing the letter of each graph alongside the correct equation.

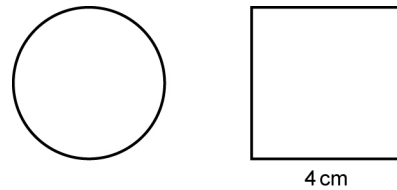
Equation	Graph
$y = x^2 + 2x - 1$	
$y = x^3 - 3x$	
$y = 3x - x^3$	
$y = 4x - x^2$	
$y = \frac{4}{x}$	
$y = x^2 - 2$	



## Y10 Higher essential skills

- 1 Geoff calculated that the mean age of the members of his badminton club was 16 years 8 months, and the range of their ages was 2 years 1 month. A new member, aged 14 years 10 months, joins the club.
- a Will the mean age of the members increase, decrease, stay the same, or is it impossible to tell?  
Explain your answer.
- b Will the range of ages increase, decrease, stay the same, or is it impossible to tell?  
Explain your answer.

- 2 The circumference of the circle and the perimeter of the square are equal. Calculate the radius of the circle. Show your method.



- 3 Show that  $3^2 + 2^3 = (3^2)^2 - 4^3$
- 4 Look at these expressions.

$$6y - 4$$

First  
expression

$$2y + 3$$

Second  
expression

What value of  $y$  makes the first expression **twice** as great as the second expression?  
Show your working.

- 5 Holly wrote the following:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p+q}$$

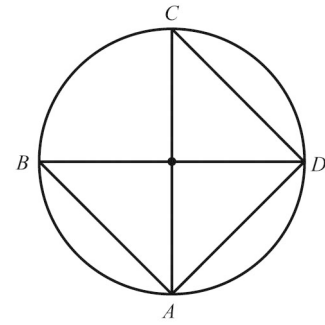
Show that Holly's statement is not correct.

- 6 I fill a glass with orange juice and lemonade in the ratio 1 : 5.  
I drink  $\frac{1}{5}$  of the contents of the glass, then I fill the glass using orange juice.

Now what is the ratio of orange juice to lemonade in the glass?  
Show your working and write the ratio in its simplest form.,

- 7 A farmer keeps sheep and hens.  
He has 84 creatures altogether.  
Between them they have 288 legs.  
Work out how many sheep and how many hens the farmer has.  
Show your method.
- 8 A student wrote 'For all numbers,  $(p + q)^2 = p^2 + q^2$ '  
Show that the student is wrong.  
Could  $(p + q)^2$  ever be the same as  $p^2 + q^2$ ?  
Explain your answer.

- 9 The diagram shows a circle with diameters  $AC$  and  $BD$ .



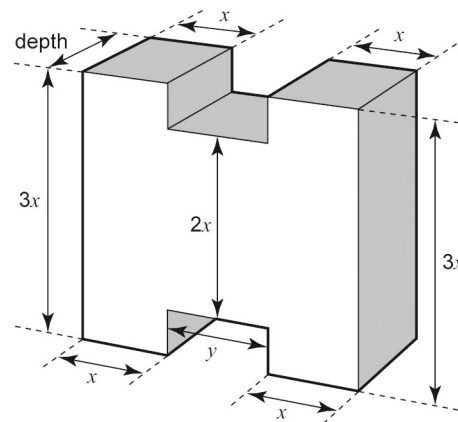
Prove that triangle  $ABD$  is congruent to triangle  $DCA$ .  
Explain your method clearly.

- 10 The difference between two numbers is 4.  
The difference between the squares of these two numbers is also 4.  
Use an algebraic method to find a pair of numbers for which these statements are true.
- 11 a Each side of a square is increased by 5%.  
By what percentage is the area of the square increased?
- b The length of a rectangle is increased by 10%.  
The width is decreased by 10%.  
By what percentage is the area of the rectangle changed?
- c A 20% increase followed by another 20% increase is not the same as a total increase of 40%.  
What is the total percentage increase?  
Show your working.
- 12 The lowest of four consecutive numbers is  $n$ .
- a Prove that there are only two sets of four consecutive numbers where the sum of the four numbers is equal to the product of the highest and lowest numbers.
- b Write down the two sets of four consecutive numbers.

- 13 The diagram shows a prism.

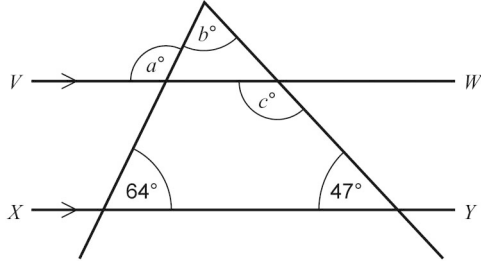
The volume of the prism is  $12x^3 - 2x^2y - 2xy^2$

Show that the depth of the prism is  $2x - y$



# Y10 Higher Geometry

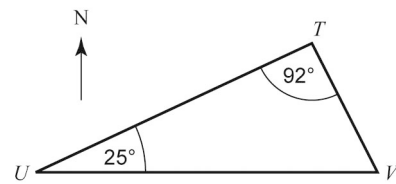
- 1 The lines  $VW$  and  $XY$  are parallel. Find the values of angles  $a$ ,  $b$  and  $c$ .



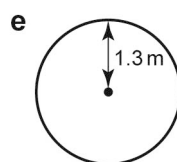
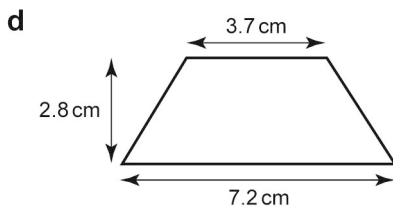
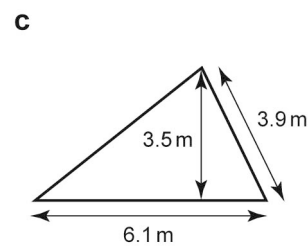
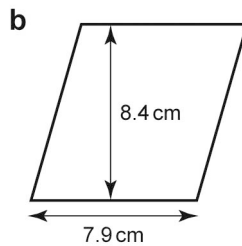
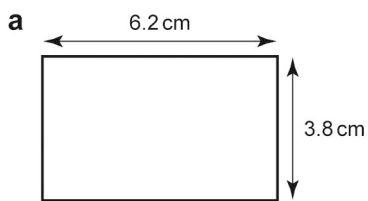
- 2 The diagram shows three airports  $U$ ,  $T$  and  $V$ .  $V$  is due east of  $U$ .

Angle  $VUT$  is  $25^\circ$  and angle  $UTV$  is  $92^\circ$ .

- What is the bearing of  $T$  from  $U$ ?
- Calculate the angle  $UVT$ . Show your working.
- Calculate the bearing of  $T$  from  $V$ .

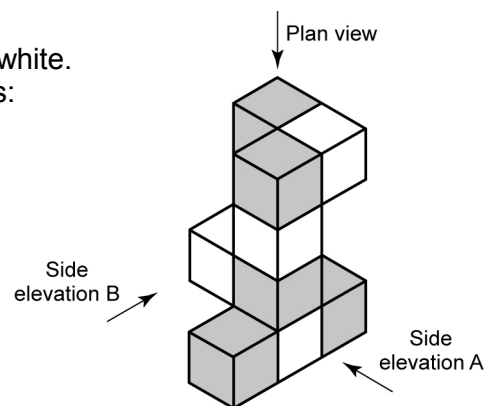


- 3 Find the area of each of these shapes.



- 4 The diagram shows a model made with nine cubes. Five of the cubes are grey. The other four cubes are white. Draw each of the following, shading the correct cubes:

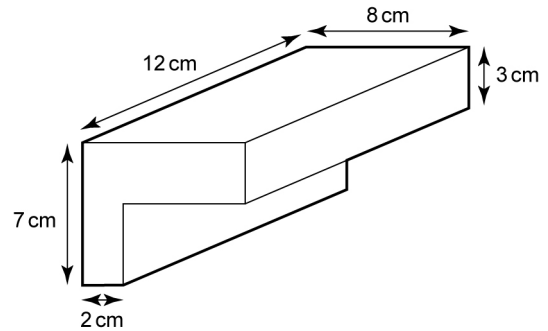
- the side elevation A
- the side elevation B
- the plan view.



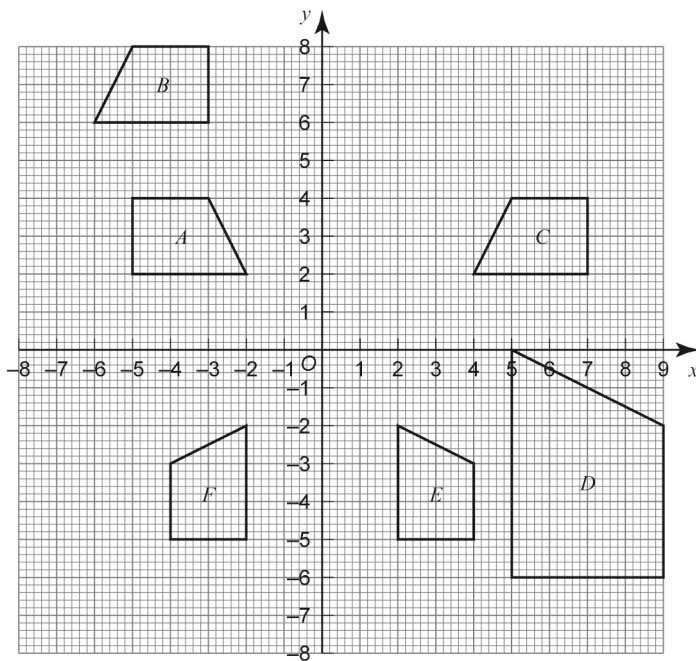
- 5 Calculate:
- the exterior angle of a regular octagon
  - the sum of the interior angles of a decagon
  - the interior angle of a regular 15-sided polygon.

- 6 The diagram shows a prism with a cross-section in an L-shape.  
Find:

- the area of the L-shaped cross-section
- the volume of the prism
- the surface area of the prism.

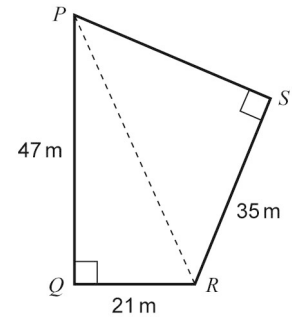


- 7 Describe fully the transformation that maps shape:
- $A$  on to  $F$
  - $B$  on to  $C$
  - $E$  on to  $D$
  - $E$  on to  $A$
  - $A$  on to  $C$ .



- 8 a Construct a triangle  $ABC$  with  $AB = 11$  cm,  $AC = 8$  cm and  $BC = 9.5$  cm  
 b Construct the locus of points 5 cm from  $A$ .  
 c Construct the locus of points equidistant from  $BA$  and  $BC$ .  
 d Shade the area inside triangle  $ABC$  that is less than 5 cm from  $A$  and nearer to  $AB$  than  $BC$ .

- 9 The diagram shows a park  $PQRS$ .  
 $PQ$  is 47 m long.  
 $QR$  is 21 m long.  
 $RS$  is 35 m long.  
 Angles  $PQR$  and  $RSP$  are right angles.  
 There is a path  $PR$  running across the park.

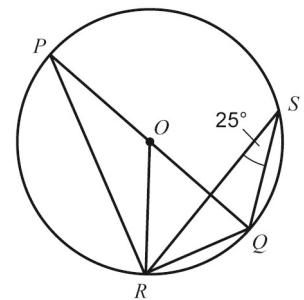


- a Calculate the length of the path,  $PR$ .  
 b Calculate the length of the side of the park,  $PS$ .

- 10 In the diagram,  $PQ$  is a diameter of the circle and  $O$  is the centre.  
 Calculate the size of angles

- a  $ROQ$     b  $RPQ$     c  $PRQ$

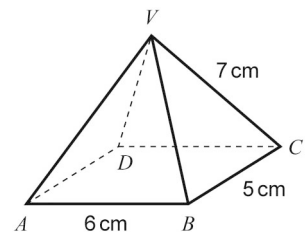
Give a reason for each of your answers.



- 11 The base of a pyramid, vertex  $V$ , is a rectangle  $ABCD$ .  
 The rectangle measures 6 cm  $\times$  5 cm  
 The length of a slant edge of the pyramid is 7 cm.

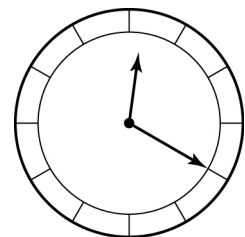
Calculate:

- a the height of the pyramid  
 b the volume of the pyramid  
 c the angle which the slant edge  $AV$  makes with the base.



- 12 The diagram shows a church clock at 12:20.  
 The hour hand is 0.8 m long and the minute hand is 1.3 m long.

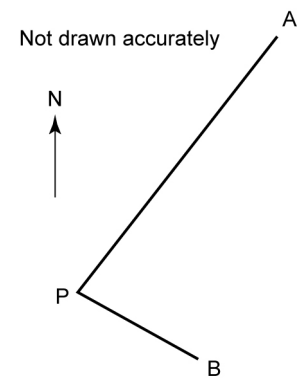
- a Calculate the angle that the hour hand has moved through since 12:00.  
 b Calculate the length of the arc, in cm, swept out by the tip of the hour hand since 12:00.  
 c Calculate the area, in  $\text{cm}^2$ , swept out by the hour hand since 12:00.  
 d Calculate the distance, in m, between the tips of the hour hand and the minute hand at 12:20.





**13** A ship,  $A$ , is 11.3 km from a port  $P$ , on a bearing of  $038^\circ$ . Another ship,  $B$ , is 4.8 km from  $P$  on a bearing of  $119^\circ$ .

- Calculate the distance,  $AB$ , between the two ships.
- Calculate the bearing of ship  $A$  from ship  $B$ .  
Give your answer to the nearest degree.



**14**  $PQRS$  is a trapezium with  $PQ$  parallel to  $SR$ .  
 $PQRT$  is a parallelogram.  
 $TR$  is twice the length of  $ST$ .

Given that  $PQ = a$  and  $QR = b$ ,  
express each of the following in terms of vectors  $a$  and  $b$ .

- $PT$
- $PR$
- $RT$
- $SR$
- $PS$

