Contents

Chapter 1: Coordinate geometry

1.1 Coordinates

Concept Corner

Use the words in the box to fill in the gaps below:

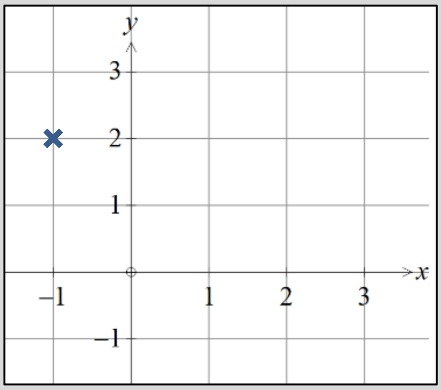
horizontal	diagonal	centre
origin	vertical	

Coordinates identify points on sets of axes.

The x-axis is ....

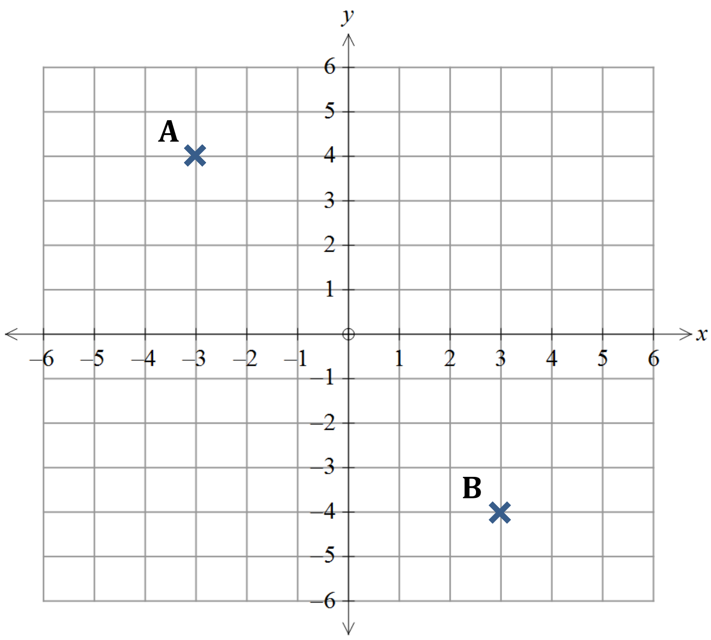
The y-axis is ....

Coordinates are written alphabetically in the form (x, y), for example, the coordinates of the point identified to the right are (−1, 2).



The point with coordinates (0, 0) is also known as the ....

1. Match the points to the correct coordinates.



Point A

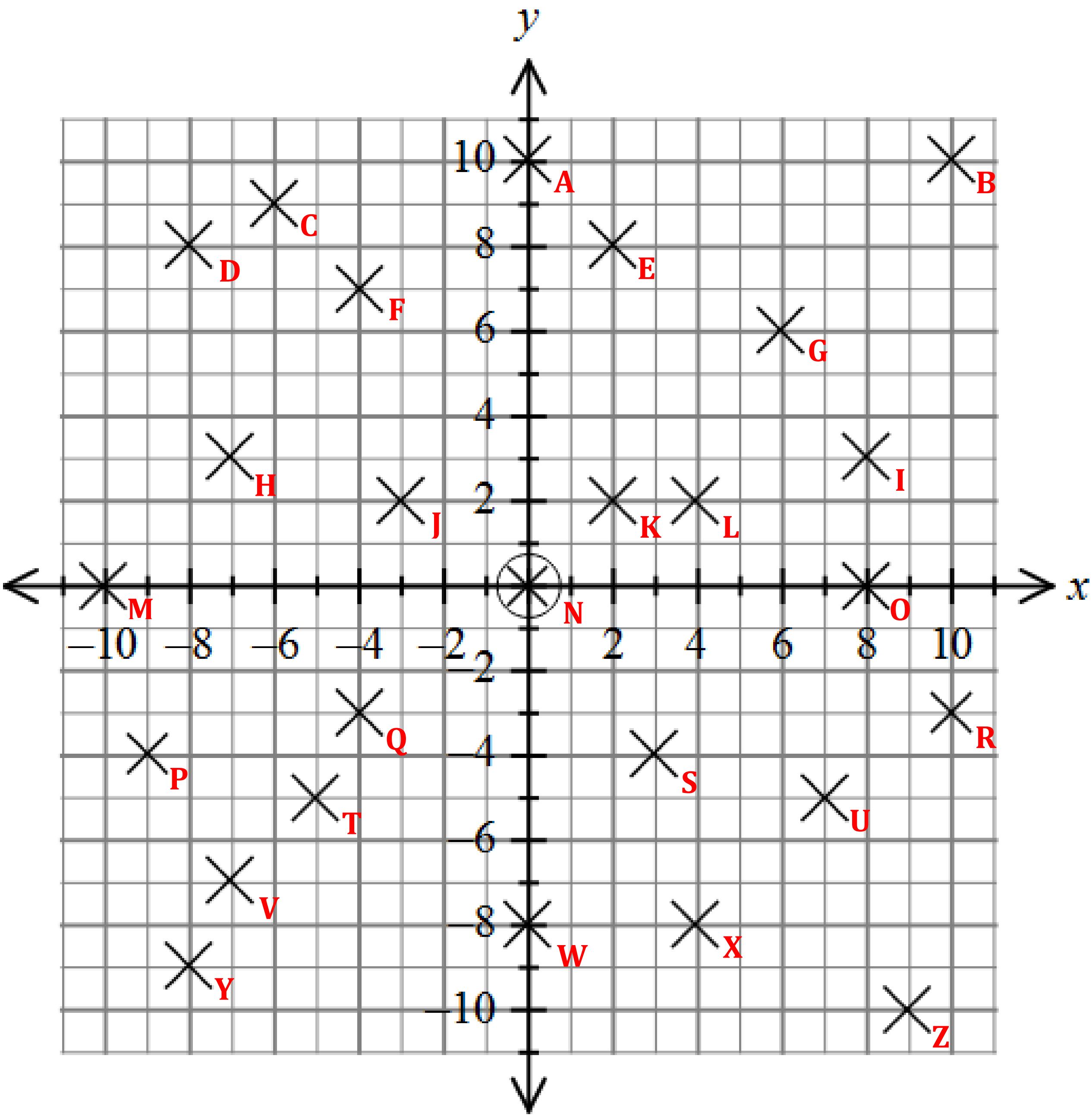
Point B

(3, 4)

(3, −4)

(−3, 4)

(−3, −4)



2. Use the information above to de-code the following sentence:

(0, −8)	(−7, 3)	(0, 10)	(−5, −5)

(−8, 8)	(8, 3)	(−8, 8)

(−5, −5)	(−7, 3)	(2, 8)

(9, −10)	(2, 8)	(10, −3)	(8, 0)

(3, −4)	(0, 10)	(−8, −9)

(−5, −5)	(8, 0)

(−5, −5)	(−7, 3)	(2, 8)

(2, 8)	(8, 3)	(6, 6)	(−7, 3)	(−5, −5)	?
					?

(0, 0)	(8, 3)	(−6, 9)	(2, 8)

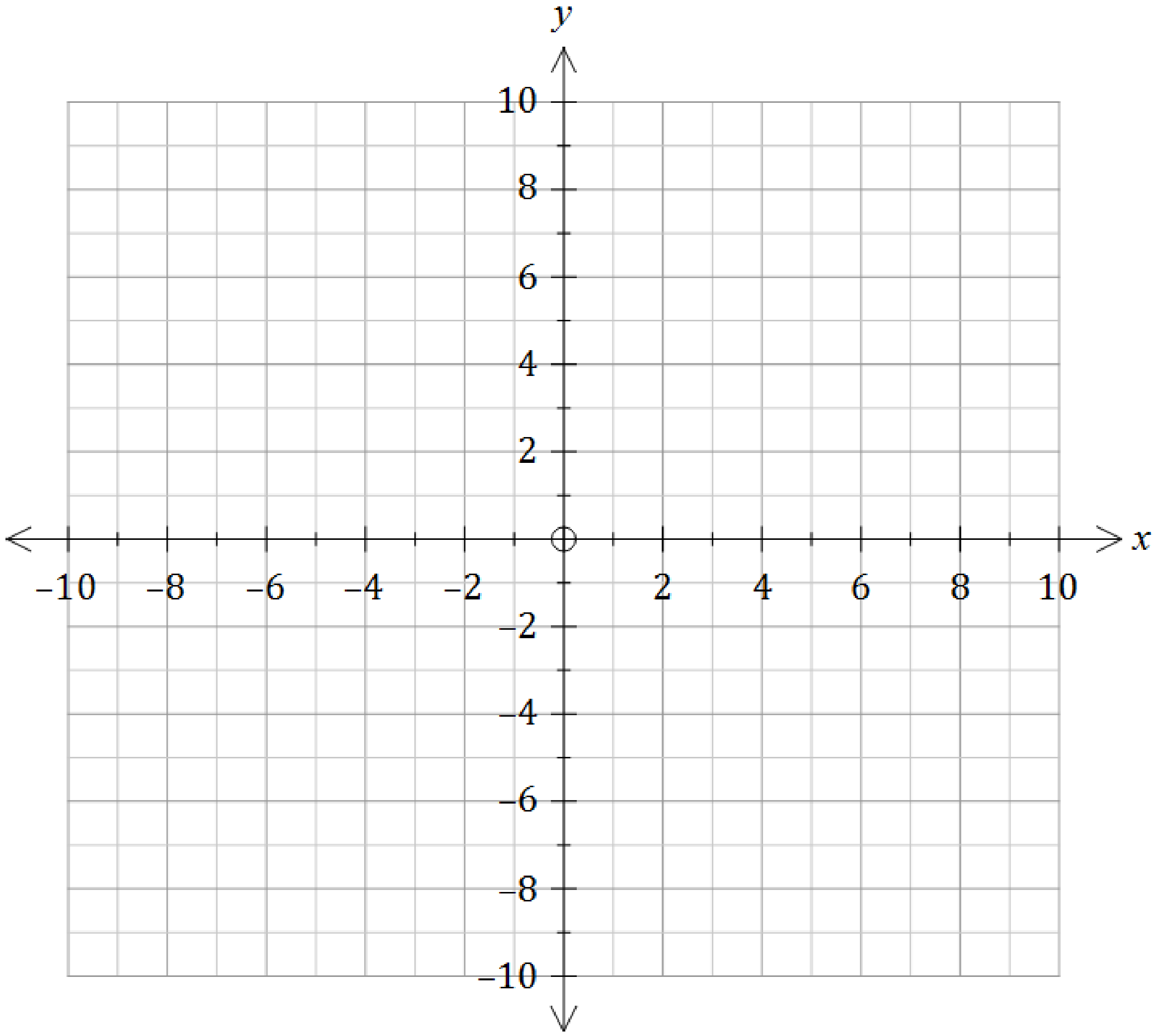
(10, 10)	(2, 8)	(4, 2)	(−5, −5)

3. Decide whether the statements below are true or false.
   a) The y-coordinate of the point (1, 7) is 1. True / False

   b) The point (0, 0) is also called the origin. True / False

   c) The vertical line that runs through (0, 0) is called the y-axis. True / False

4.



   a) Plot the following points and write down the coordinates of the fourth point to make a rectangle.

      (6, 3), (−1, 3), (6, 5), (………, ………)

   b) Plot the following points and write down the coordinates of the fourth point to make a square.

      (− 4, 1), (− 7, 4), (− 1, 4), (………, ………)

   c) Plot the following points and write down the coordinates of a fourth point to make a parallelogram.
      (− 3, − 2), (− 1, − 5), (− 7, − 6), (………, ………)

d) The points (4, −3) and (5, −9) form two vertices of an isosceles triangle.
Find three different coordinate pairs which could each represent the third vertex.

(………, ………) (………, ………) (………, ………)

e) Calculate the area of each of the quadrilaterals that you have drawn on the axes above.

Area of rectangle =

Area of square =

Area of parallelogram =

5. Decide whether the statements below are true or false.
Drawing a set of axes with a suitable scale may help.

a) The points (0, 0), (0, 120) and (40, 0) will form a right-angled triangle.
True / False

b) The points (4, 1), (8, 4) and (6, 7) will form an isosceles triangle.
True / False

c) The square whose vertices have coordinates (−3, 5), (−2, 2), (1, 3), (0, 6) has an area of 9 square units.
True / False

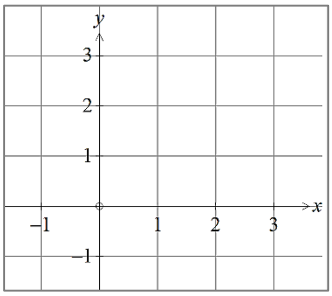
6. I am thinking of a point on the axes below whose coordinates have integer values.
Use the three clues below to identify the coordinates of the point.

My x-coordinate is odd.

The x-coordinate is more than the y-coordinate

My y-coordinate is the first prime number

Answer: (………, ………)

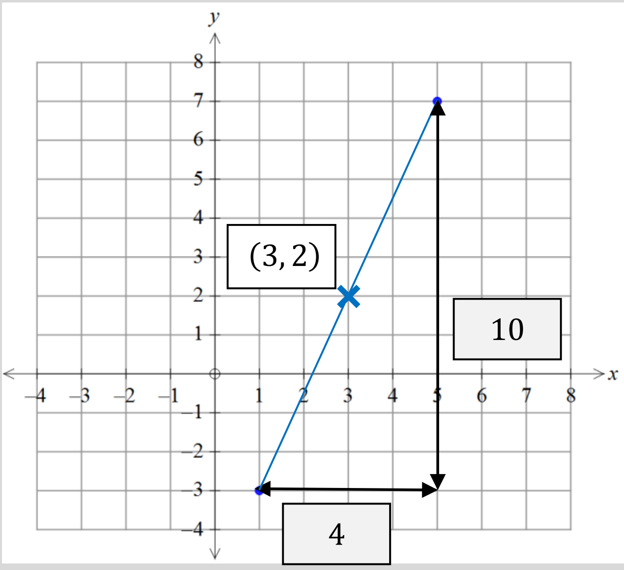


1.2 Midpoint of a line

Concept Corner

Sarah and Katherine have used two different methods to find the midpoint of the line segment joining the points (1, –3) and (5, 7).

Sarah



Katherine

$$\left(\frac{1+5}{2}, \frac{-3+7}{2}\right)$$

So the midpoint

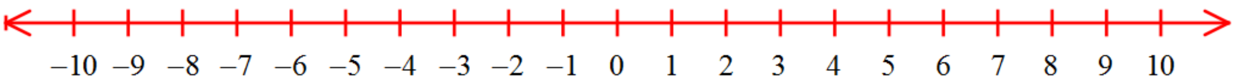
is (3, 2)

Explain what each student has done.

....

....

....



1. Write down the number which lies half-way between:

a) 0 and 8 ....

b) 8 and 10 ....

c) 3 and 8 ....

d) −8 and 8 ....

e) −2 and 8 ....

f) −1 and −8 ....

g) −7.3 and 8.0 ....

h) −8.75 and $8\frac{1}{4}$ ....

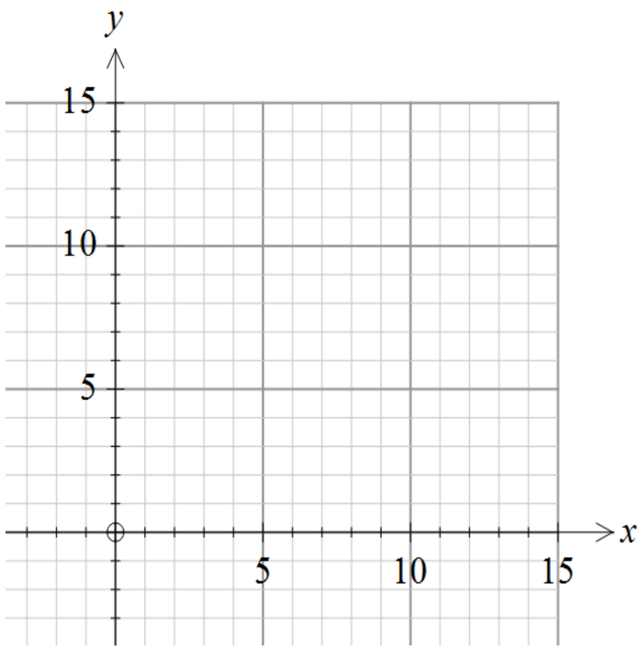
2. a) Tick the appropriate column(s) to indicate whether the following line segments will cross the x-axis, y-axis, or neither.

	x-axis	y-axis	Neither
$(2, 6)$ and $(14, 12)$			
$(-2, -2)$ and $(-4, 3)$			
$(-4, -14)$ and $(-2, 4)$			
$(4, -2)$ and $(7, -2)$			
$(-5, 6)$ and $(11, 7)$			

b) Write down the coordinates of the end-points of a line segment which crosses **both** the x-axis and the y-axis.

(…………, …………) and (…………, …………)

3. Calculate the midpoint of each of the following line segments.
You may use the axes below if needed.



a) (4, 0) and (8, 0)

Midpoint (…………, …………)

b) (7, 5) and (7, 13)

Midpoint (…………, …………)

c) (10, 7) and (14, 11)

Midpoint (…………, …………)

d) (74, 28) and (31, 35)

Midpoint (…………, …………)

You may use the axes below to help with the following questions if needed.

4. The midpoint of line segment AB has coordinates (3, 5).
   The coordinates of point A are (0, 5).
   Write down the coordinates of point B.

(………, ………)

5. The midpoint of line segment PQ has coordinates (3, 5).
   The coordinates of point P are (5, 7).
   Write down the coordinates of point Q.

(………, ………)

6. Calculate the midpoint of each of the following line segments using the axes below if necessary.

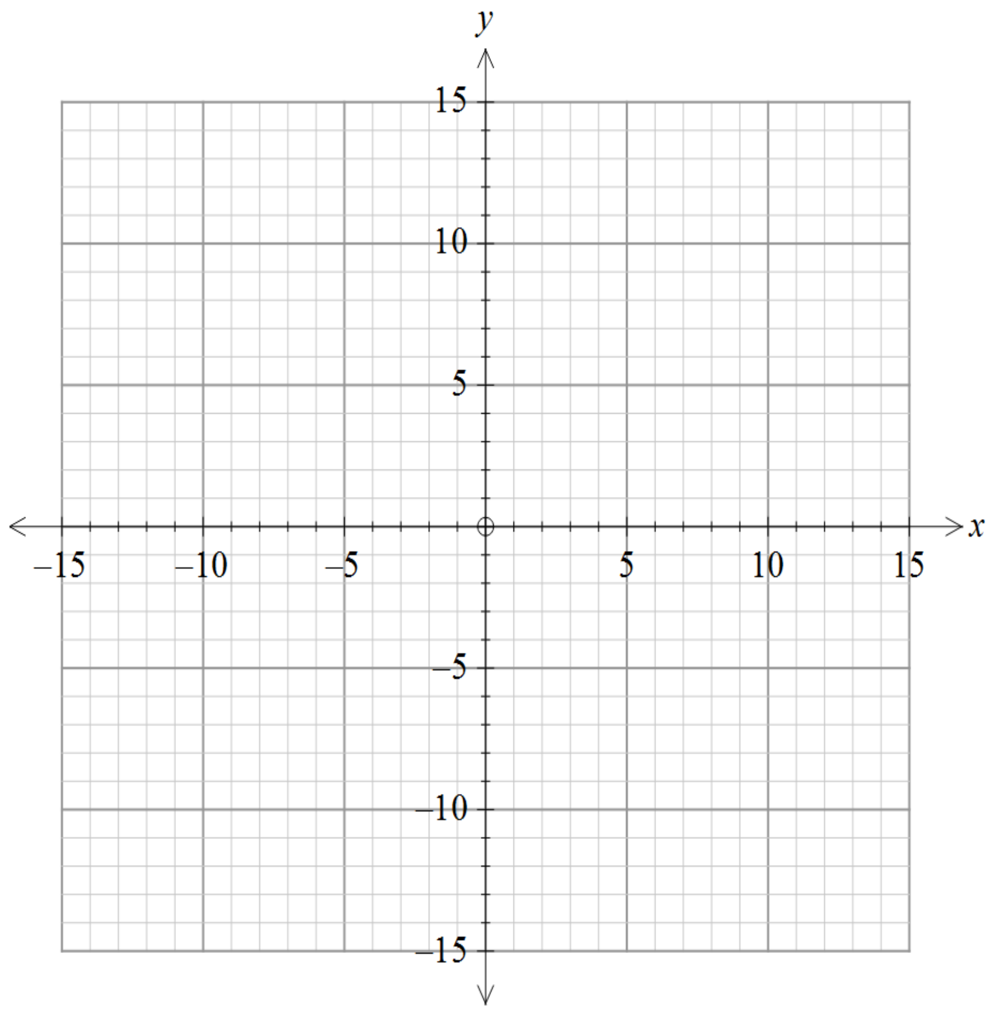
   a) (−8, 4) and (−6, 5) Midpoint (………, ………)

   b) (10, −4) and (−5, 15) Midpoint (………, ………)

   c) (−2.4, −1.5) and (−6, 4.5) Midpoint (………, ………)

   d) $(16a, -24b)$ and $(2a, 0)$ Midpoint (………, ………)

   e) $(8a, b)$ and $(-9a, 3b)$ Midpoint (………, ………)



7. Solve the following problems, using the axes below if required.

a) The midpoint of a line segment AB is point M.
The coordinates of point M are $(5, -3)$ and the coordinates of point A are $(2, 0)$.
What are the coordinates of point B?

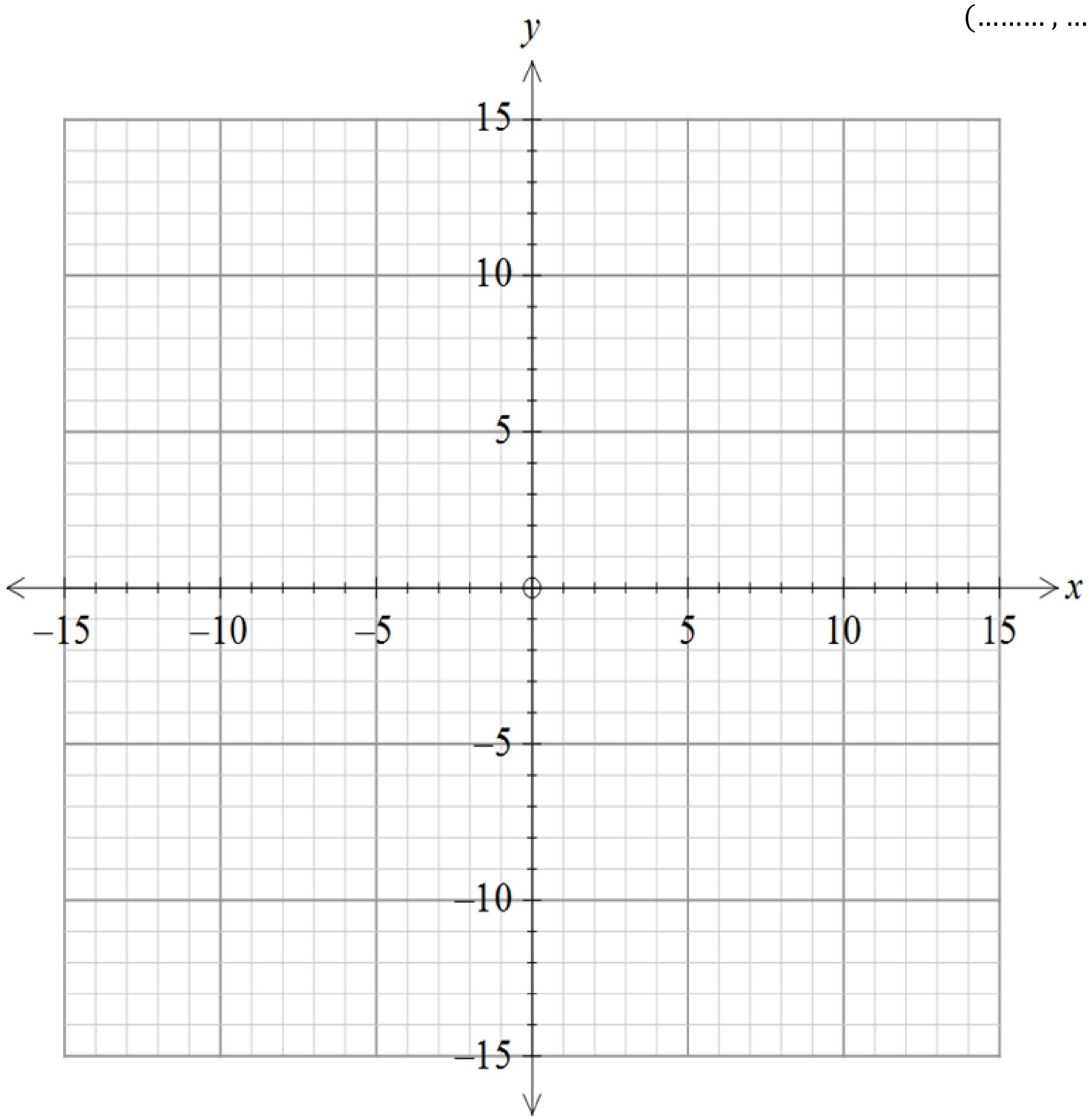
(……… , ………)

b) The line segment joining $(-4, 10)$ and $(7, 3)$ forms the diameter of a circle.
What are the coordinates of the centre of the circle?

(……… , ………)

c) The centre of a circle has coordinates $(3, 4)$. The point $X(-1, 9)$ lies on the circumference of the circle. If the line segment XY forms a diameter, find the coordinates of point Y.

(……… , ………)



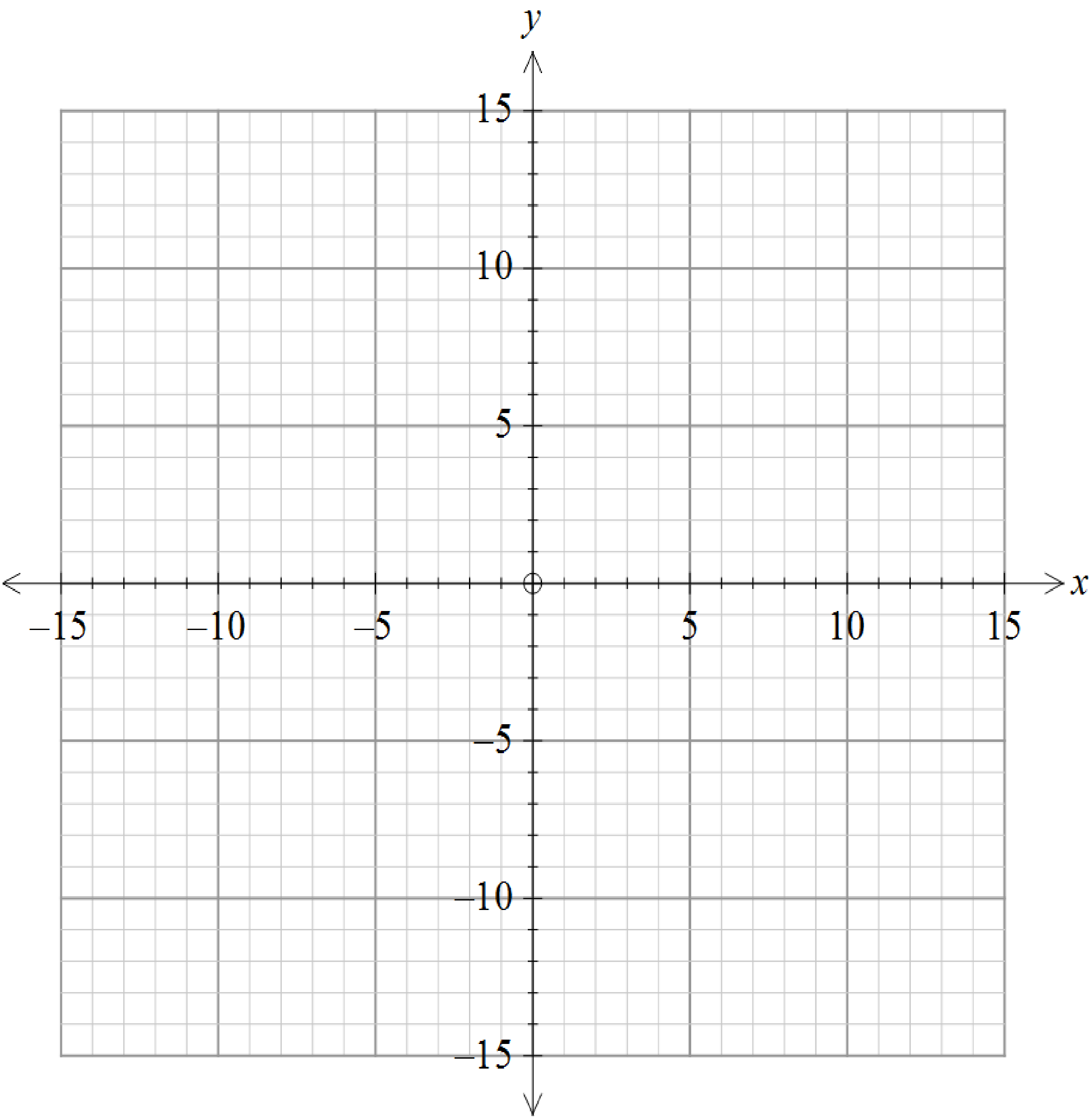
8. Solve the following problems, using the axes below if required.

a) If the point $(x, 10)$ is the midpoint of the points $(5, 7)$ and $(-9, y)$, calculate the values of x and y.

x = ………, y = ………

b) The points A(−4, 14) and B(4, −6) form the line segment AB.
The point P lies on the line segment AB.
The lengths of AP and PB are in the ratio 1 : 3.
What are the coordinates of point P?

(……… , ………)



Reflections

This space is for you to write your reflections on the whole unit on coordinates.

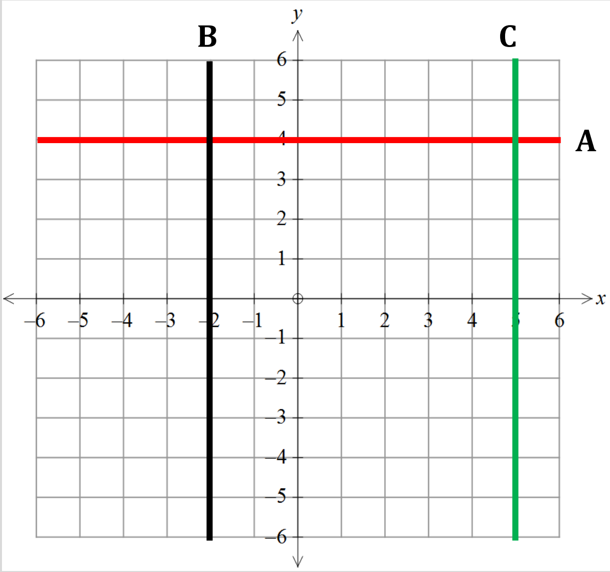
You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

Chapter 2: Linear graphs

2.1 Horizontal and vertical lines

Concept Corner



The graph to the right shows three straight lines.

Write down the coordinates of three points which lie on line A:

……………………………………………………

Why is the equation of this line $y = 4$?

……………………………………………………

……………………………………………………

Write down the coordinates of three points which lie on line B:

……………………………………………………

Why is the equation of this line $x = -2$?

………

Write down the equation of line C.

……………………………………………………

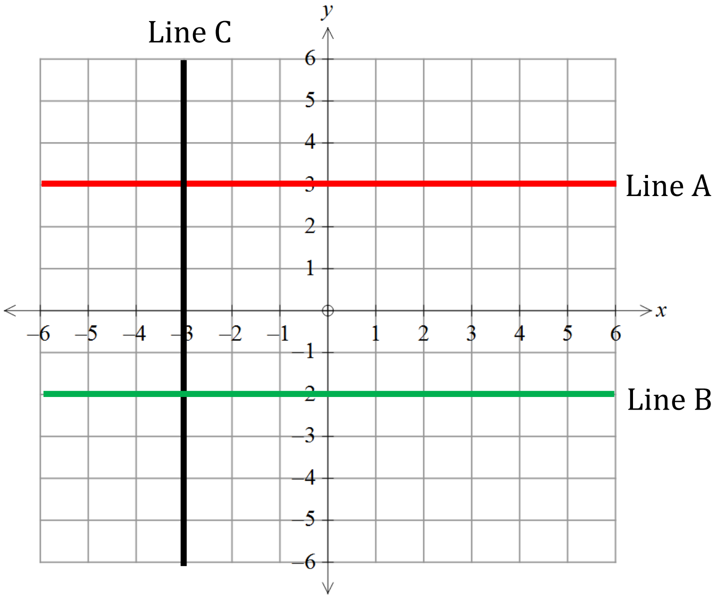
The coordinates $(4, -1), (3, -1)$ and $(-2, -1)$ all lie on a different line.
Write down the equation of this line and add it to the graph.

……………………………………………………

1. Match each equation to the correct set of points, filling in the gaps where appropriate.

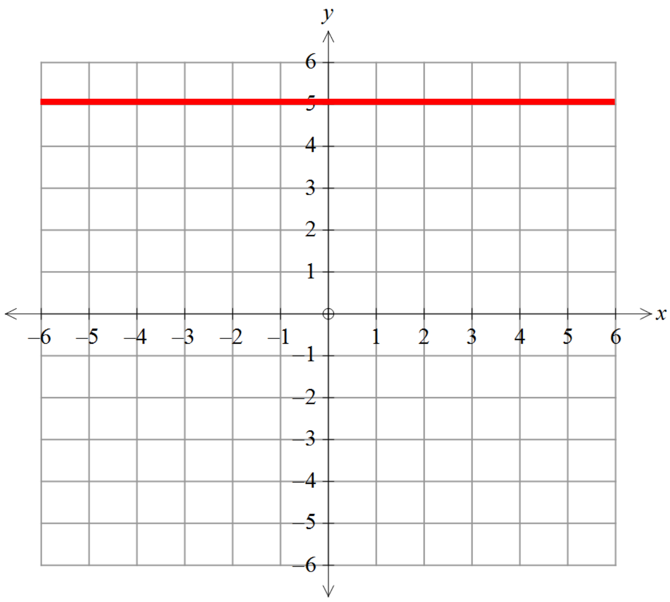
$y = 5$	$(3, 3), (3, 4), (3, 5), (3, 6)$
$x = 3$	$(-5, 2)$, (…… , 4), $(-5, -5), (-5, 8)$
$y = 3$	$(4, 5), (-7, 5)$, (3,……), $(5, 5)$
$x = -5$	$(5, -2), (5, -1)$, (5, ……), $(5, 1)$
$x = 5$	$(3, 3), (2.5, 3), (-13, 3)$, (2, ……)

2. Write the equations of the lines A to C below.
   Draw lines D to F on the same set of axes.



Line	Equation
A	
B	
C	
D	$x = 5$
E	$y = 0$
F	$x = -5$

3. Toby has tried to draw the line $x = 5$ on the axes below.
   Explain *why* he is incorrect and correct his mistake.



……

……

……

4. True or false?

a) The point $(0, 5)$ lies on the line $x = 5$ — True / False

b) The point $(7, -7)$ lies on the line $y = 7$ — True / False

c) The point $(2, 531)$ lies on the line $x = 2$ — True / False

d) The origin lies on the y-axis — True / False

e) The points $(3, 4)$, $(-7, 4)$, $(1.7, 4)$ can be joined to form a vertical line — True / False

f) The point $(9, 12)$ lies on both of the lines $x = 12$ and $y = 9$ — True / False

5. Write down:

a) The equations of two different horizontal lines.

....

b) The equation of a line perpendicular to the line $y = 9$.

....

c) The equations of four lines which will enclose a rectangle with area 18 square units.

....

d) The equations of four lines which will intersect in four different quadrants to form a square with an area of 25 square units.

....

2.2 Gradient

Concept Corner

Use the words in the box to fill in the gaps.

parallel	gradient
negative	positive

The slope or steepness of a straight line is

a measure of its ……………………………………………..............

The line $y = 2x - 3$ will have a ……………………………………………. gradient, whilst the line

$y = -2x + 3$ will have a ……………………………………………. gradient.

Lines with the same gradient will be ……………………………………………. to each other.

Explain how the diagram to the right shows that the gradient of the line is 2.

………………………………………………………………………

………………………………………………………………………

………………………………………………………………………

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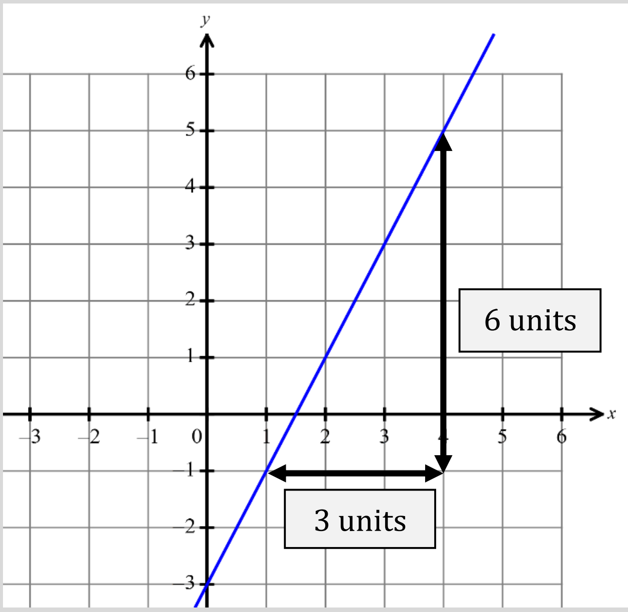
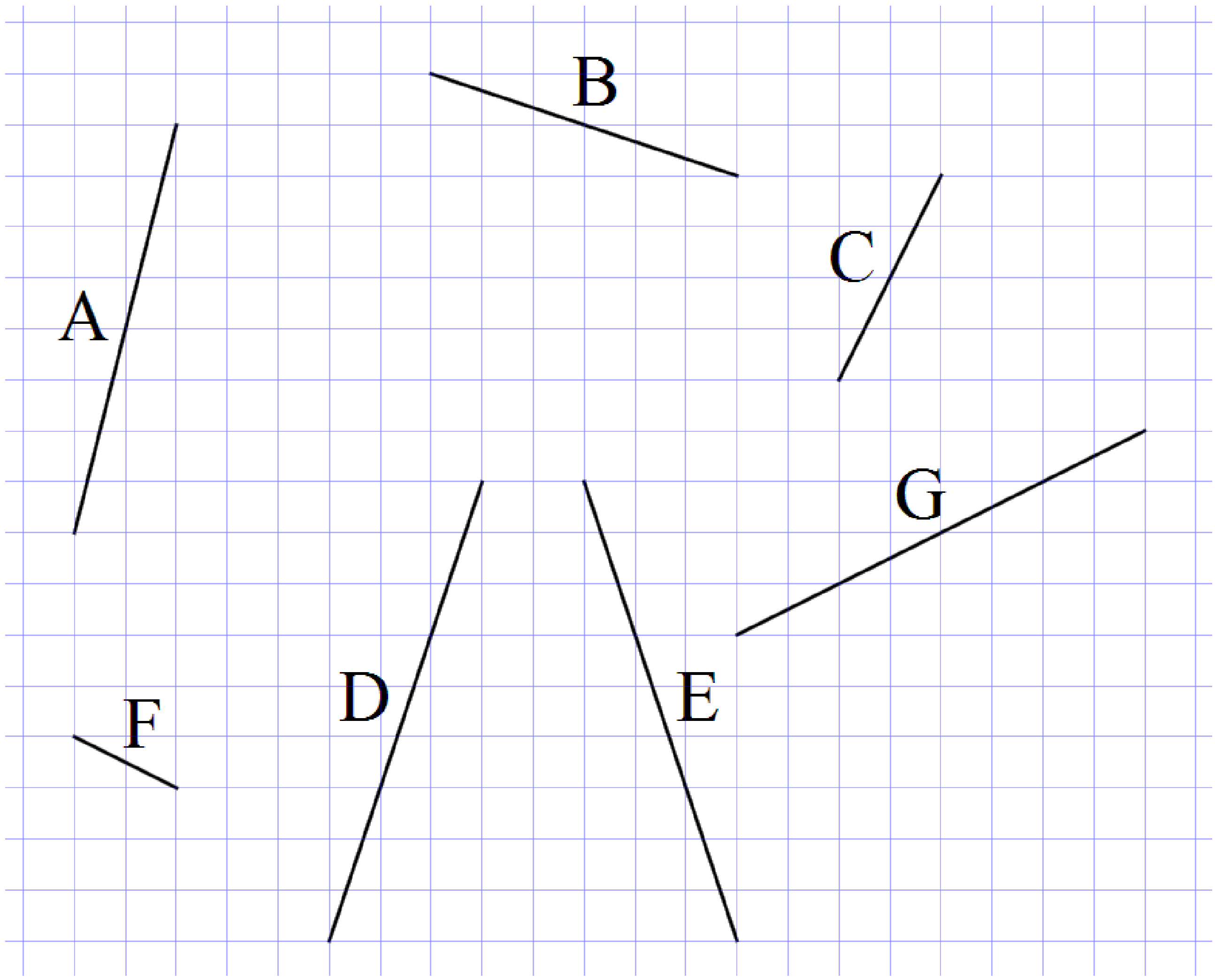
………………………………………………………………………



1. Calculate the gradients of the lines on the grid below using the values provided. On the grid, one square is one unit.

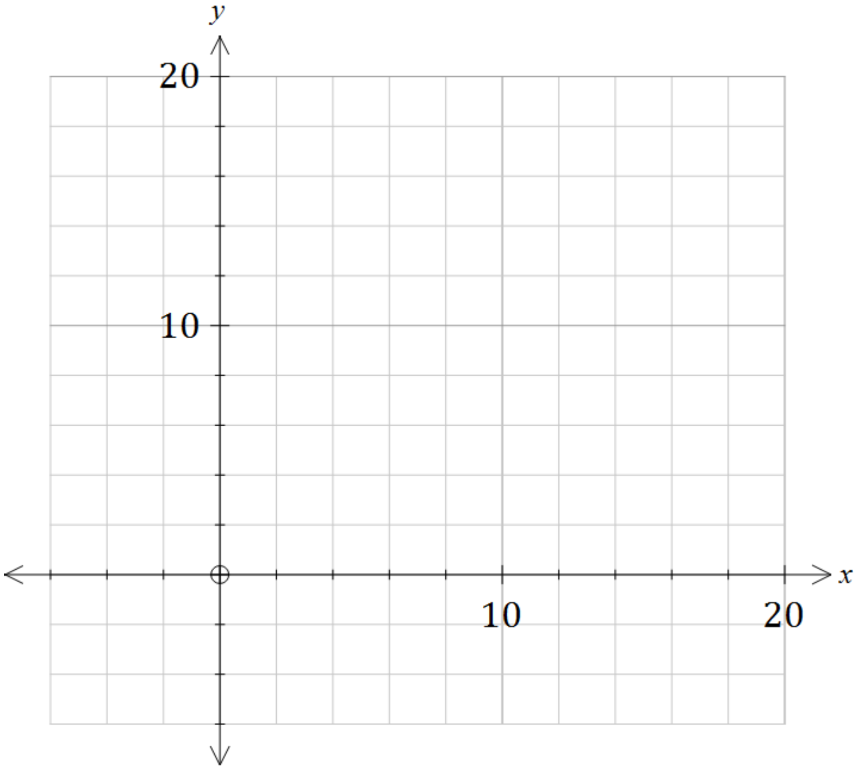
Line segment	A	B	C	D	E	F	G
Gradient							

2 3 -3 $-\frac{1}{2}$ $-\frac{1}{4}$ $\frac{1}{2}$ $-\frac{1}{3}$ 4



One of the values is left over. Draw a line segment with this gradient on the grid above.

2. Calculate the gradient of each line segments joining the points below. You may find the axes useful.



a) (2, 4) and (4, 16)

……………………………………

b) (2, 10) and (4, 6)

……………………………………

c) (0, 4) and (6, 6)

……………………………………

d) (6, 10) and (12, −2)

……………………………………

3. Write down the coordinates of the midpoints for each line segment above.

a) Midpoint (………, ………)

b) Midpoint (………, ………)

c) Midpoint (………, ………)

d) Midpoint (………, ………)

2.3 Plotting linear graphs

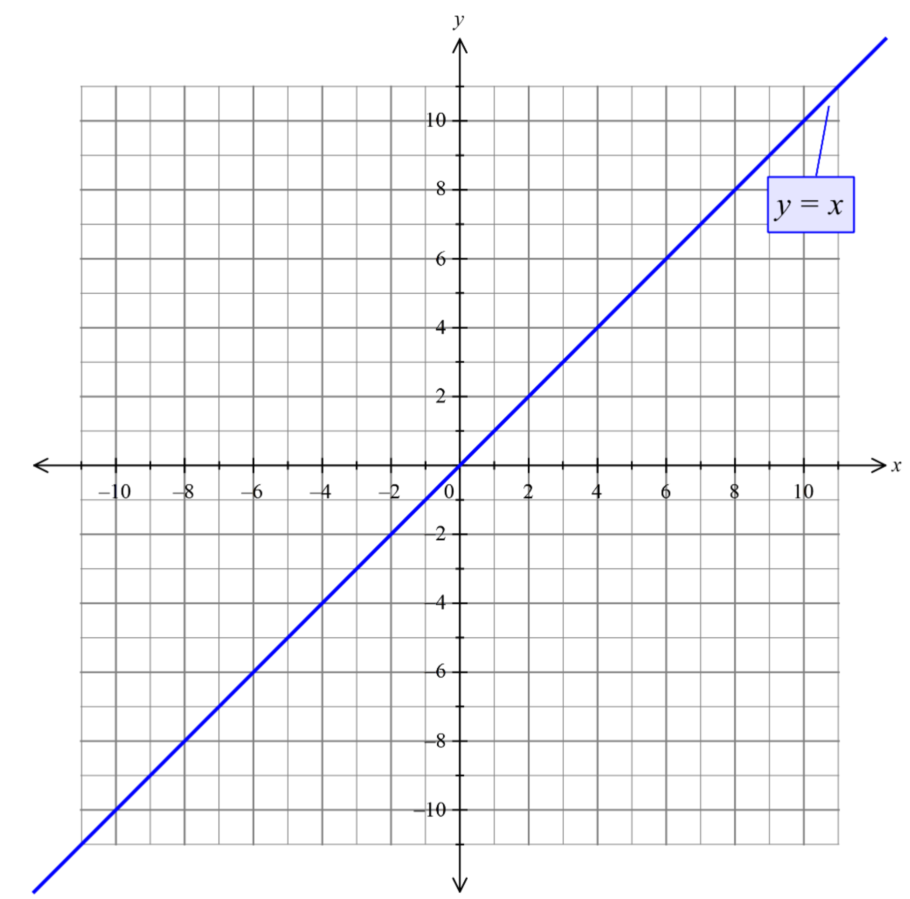
Concept Corner

The diagram below shows the graph of $\boldsymbol{y = x}$.

Write down the coordinates of four points which lie on the line $y = x$.

....

....



What do you notice about the x- and y-coordinates of these points?

....

....

....

....

The following points lie on the line $y = 2x$:

$(0, 0)$ $(1, 2)$ $(2,$ …….$)$ $($ …….., $10)$ $(-3, -6)$.

Use these points to plot the line $y = 2x$ on the axes above.

Which line is steeper, $y = x$ or $y = 2x$?

....

1. By finding the coordinates of points on each line, plot the graph of each equation on the axes below.

   a) $y = x$

   (0, ...) (.......... , ...) (..........., ...)

   b) $y = 2x$

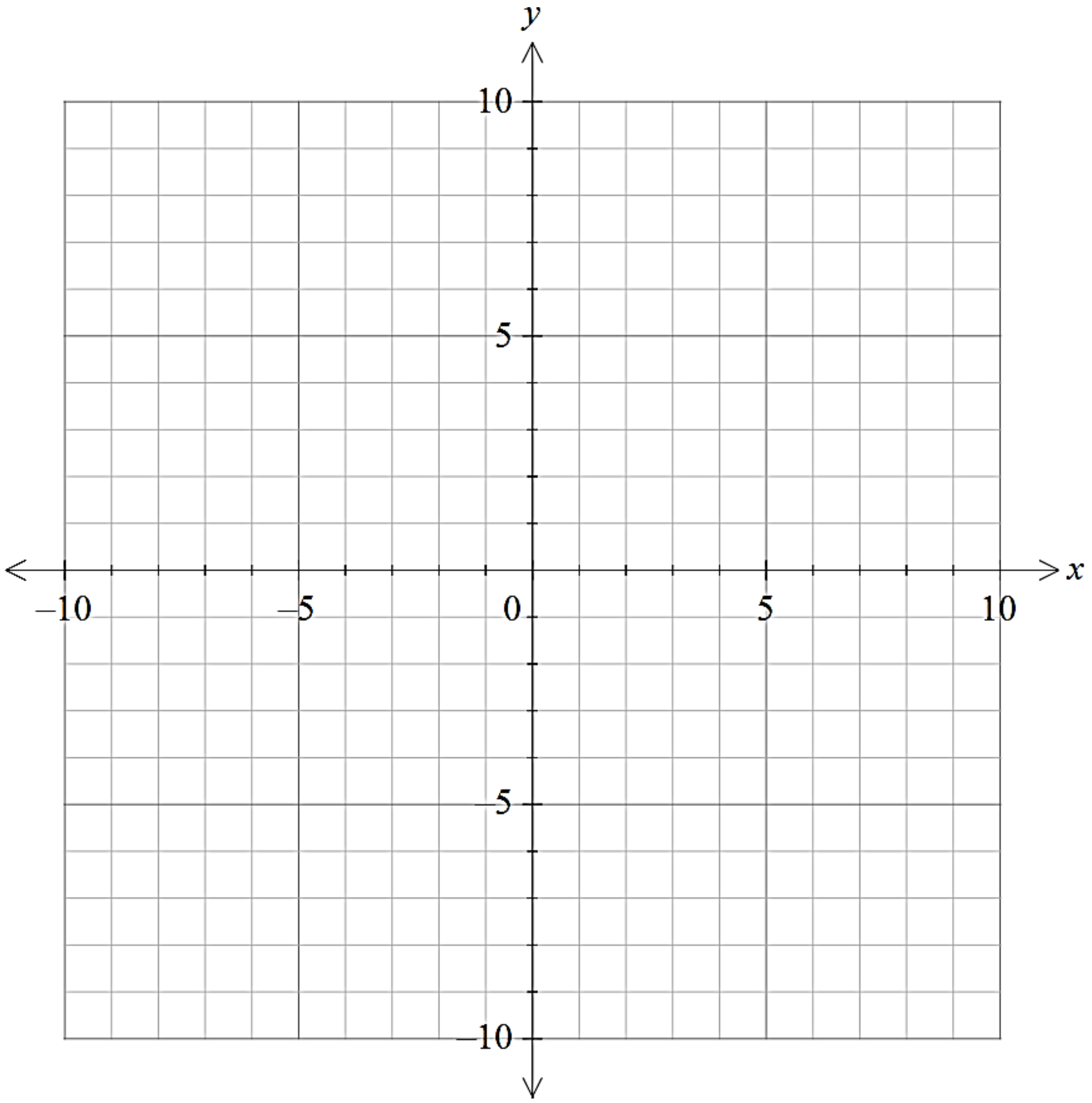
   (0, ...) (.......... , ...) (.......... , ...)

   c) $y = 2$

   (0, ...) (.......... , ...) (.......... , ...)

   d) $y = \frac{x}{2}$

   (.......... , ...) (.......... , ...) (.......... , ...)



2. By finding the coordinates of points on each line, plot the graph of each equation on the axes below.

a) $y = x + 5$

(0, ……….)　　(………. , ……….)　　(……….., ……….)

b) $y = x - 5$

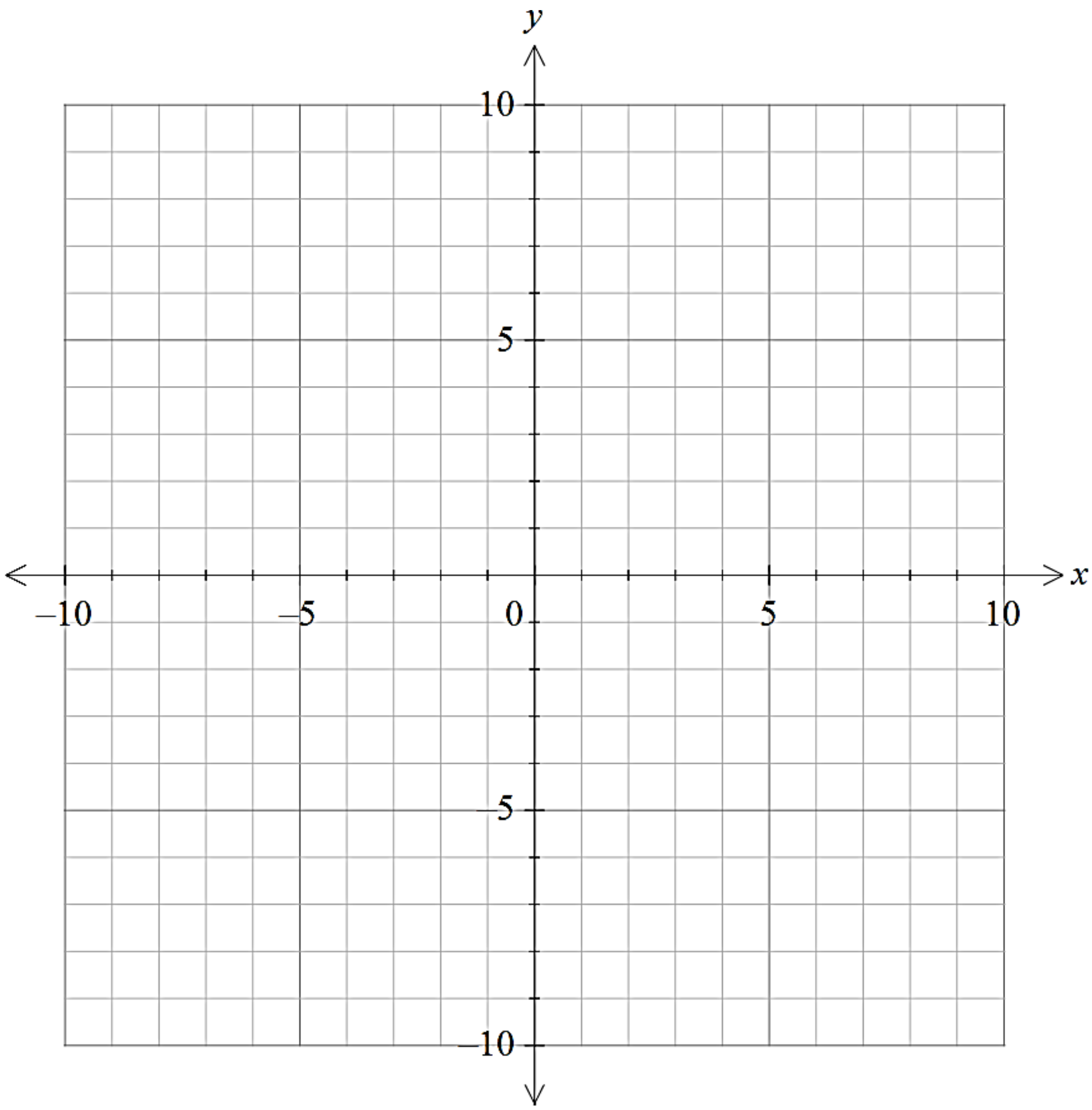
(0, ……….)　　(………. , ……….)　　(………. , ………..)

c) $y = 5 - x$

(0, ……….)　　(………. , ……….)　　(………. , ………..)

d) $y = -(x + 5)$

(………. , ……….)　　(………. , ……….)　　(………. , ………..)



3. By finding the coordinates of points on each line, plot the graph of each equation on the axes below.

a) $y = 3x$

(………. , ……….) (………. , ……….) (………. , ……….)

b) $y = 3x + 5$

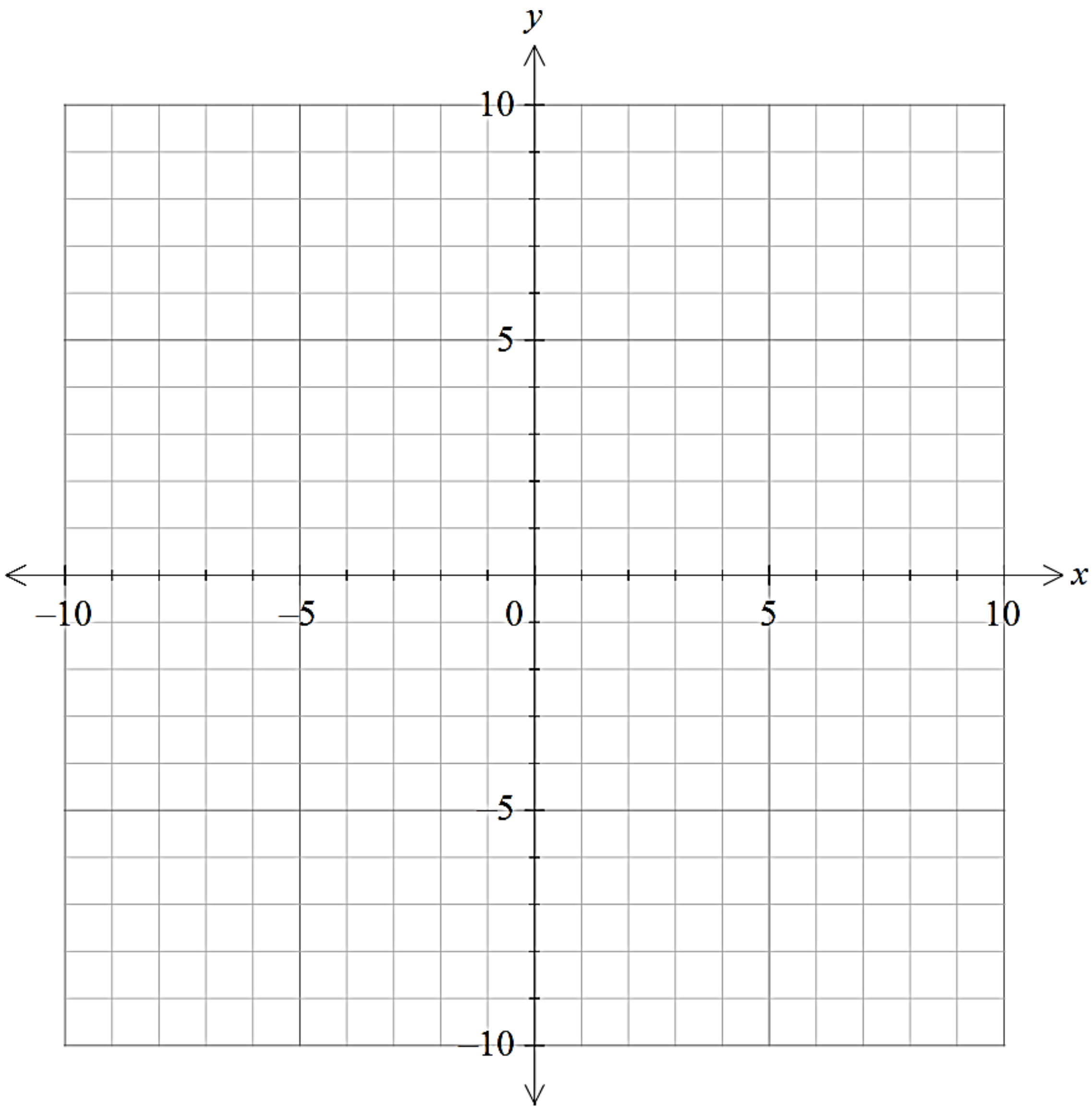
(………. , ……….) (………. , ……….) (………. , ……….)

c) $y = -3x + 5$

(………. , ……….) (………. , ……….) (………. , ……….)

d) $y = -(3x + 5)$

(………. , ……….) (………. , ……….) (………. , ……….)



4. By finding the coordinates of points on each line, plot the graph of each equation on the axes below.

a) $y = 2x + 3$

(………. , ……….) (………. , ……….) (………. , ………..)

b) $y = 3x + 2$

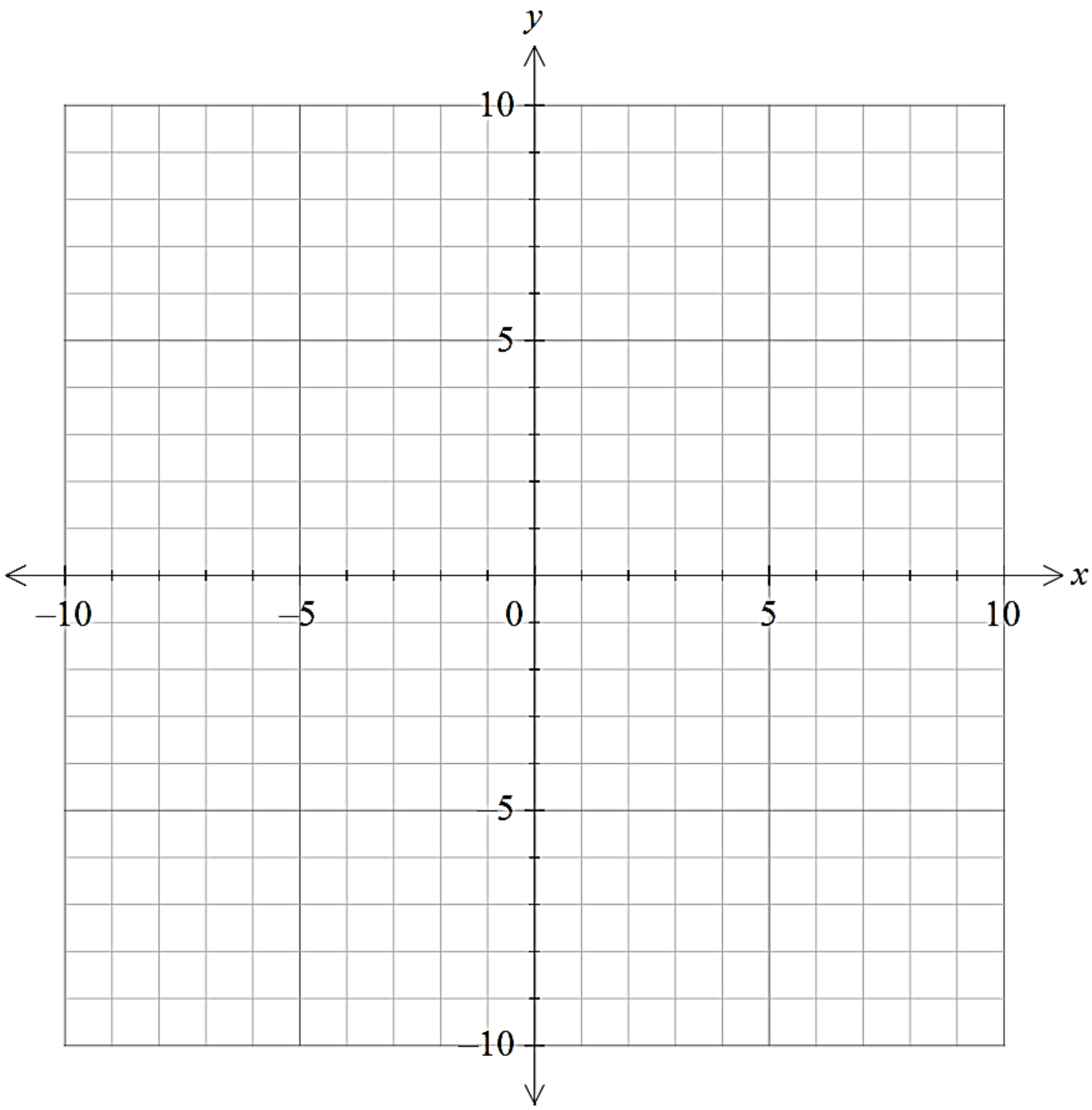
(………. , ……….) (………. , ……….) (………. , ………..)

c) $y = \frac{x}{2} - 3$

(………. , ……….) (………. , ……….) (………. , ………..)

d) $y = \frac{1}{2} - 3x$

(………. , ……….) (………. , ……….) (………. , ………..)



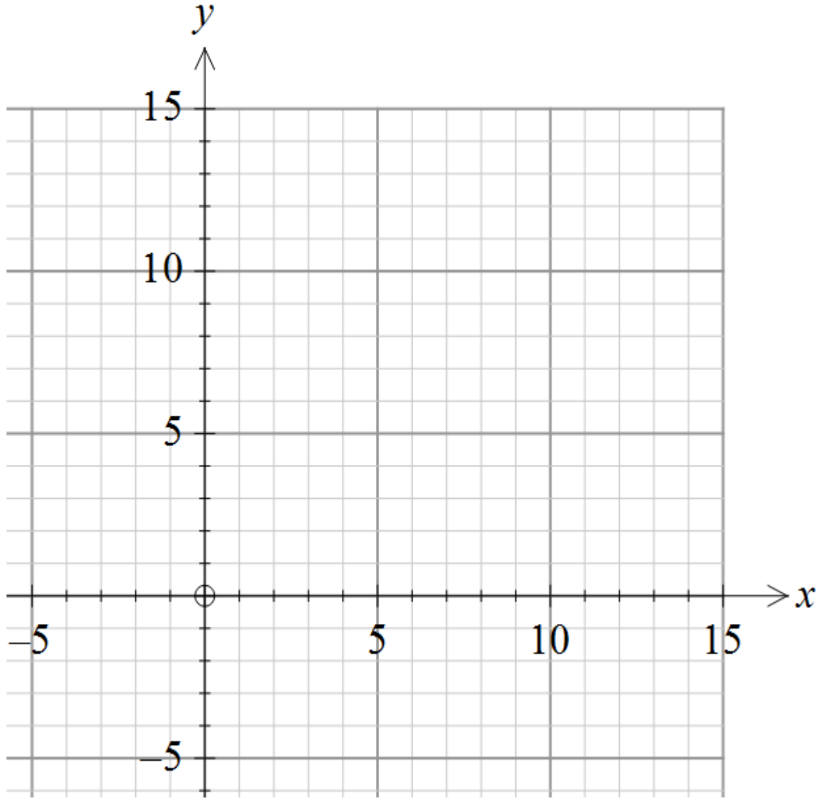
5. Leo is trying to draw the graph of $x + y = 5$ and has chosen the following points:

$(0, 5)$, $(1, 5)$, $(2,5)$.

Leo's coordinates are **wrong**.
What is the equation of the line these points lie on?

....

Work out the coordinates of three correct points and plot the graph $x + y = 5$.



6. Match the coordinates to the line that they lie on.
Some pairs of coordinates lie on more than one line.

$y = 2x$	$(102, 204)$
$y = 2x + 1$	$(-8, -7)$
$y = x + 1$	$(19, 2)$
$y = 2$	$(0, 0)$
	$(0.5, 2)$

e) True or false?

The origin lies on the line $y = 3x$

The coordinate $(3, -7)$ lies on the line $y = \frac{x}{3} - \mathbf{7}$

The coordinate $(31, 132)$ lies on the line $y = 4x - 8$

The line $x = 5$ will cross the line $y = 3x + 6$

The coordinate $(16, -67)$ lies on the line $y = 13 - 5x$

2.4 The equation of a line

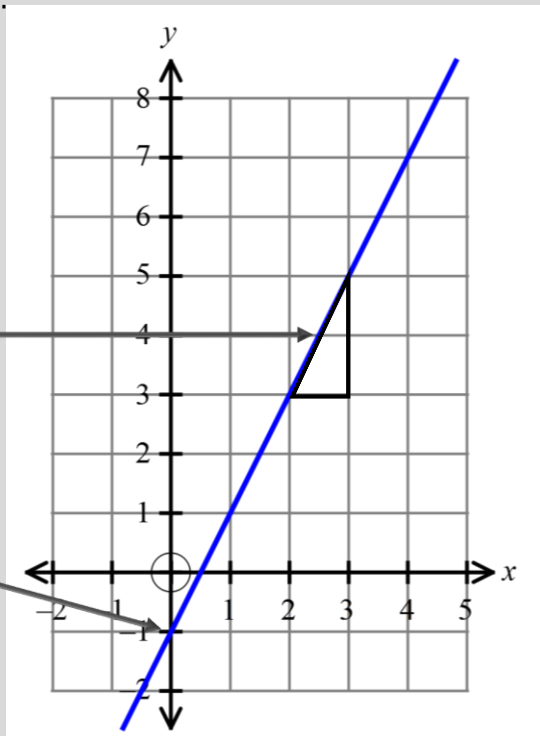
Concept Corner

The equation of a straight line can be written in the form:

$y = \boldsymbol{m}x + \boldsymbol{c}$

m is the gradient

c is the y-coordinate of the y-intercept



gradient of 2

y-intercept is $(0, -1)$

An equation of the line is therefore $\boldsymbol{y = 2x - 1}$.

1. Complete the table below for each equation.

Equation	**Gradient**	***y*-intercept**
$y = 3x$		
$y = x$		
$y = 3x - 7$		
$y = 7 + 3x$		
$y = 7 - 3x$		
$y - 7 = 3x$		
$y + 3x = 7$		

2. Some students make the statements below.
   Are they correct? If not, correct them.

The gradient of the line $y = 3x - 4$ is $3x$.

The y-intercept of the line $y = 3 - 5x$ is $(0, -5)$

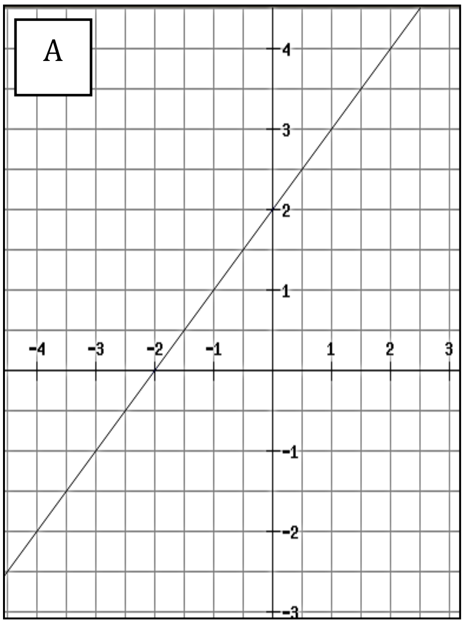
The y-intercept of the line $y = 11 - 2x$ is $(0, 2)$.

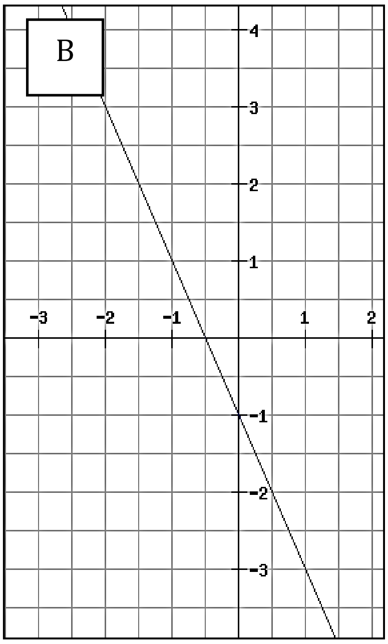
The gradient of the line $y + x = 5$ is $(0, 1)$.

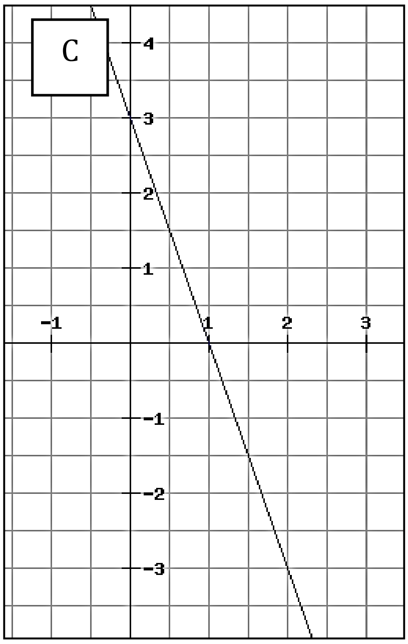
The y-intercept of the line $y = 5x$ is $(0, 0)$.

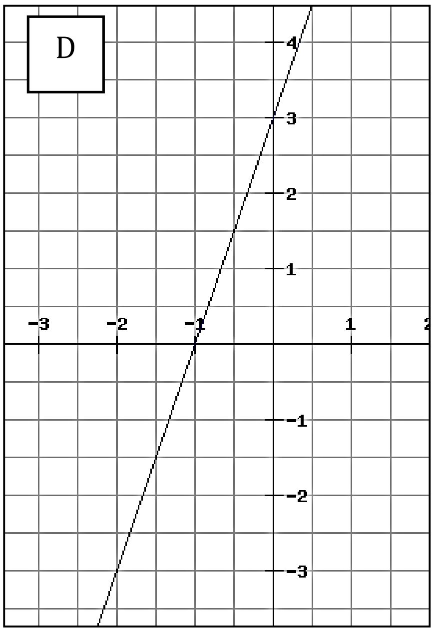
The lines $y = 0.5x$ and $y = \frac{1}{2}x - 3$ are parallel.

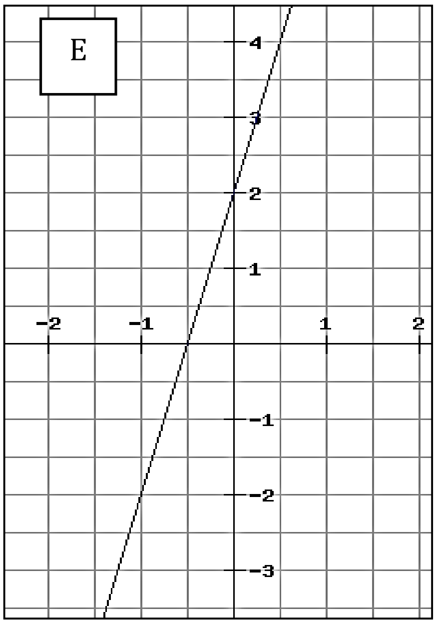
3. Match the equation of the line to the graphs below







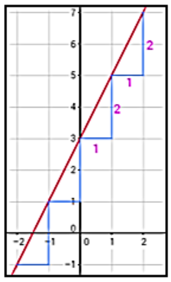




$y = 3x + 3$	$y = -3x + 3$	$y = x + 2$
$y - 2 = 4x$	$y = 4x + 2$	$y = -2x - 1$
$y = -(2x + 1)$	$y = -(3x - 3)$	$x = -y + 2$

Concept Corner: Four representations

$$y = 2x + 3$$



A straight line,
with gradient 2,
and y-intercept (0,3).

+1 +1 +1

x	0	1	2	3
y	3	5	7	9

+2 +2 +2

Explain how each represents the same mathematics in different ways.

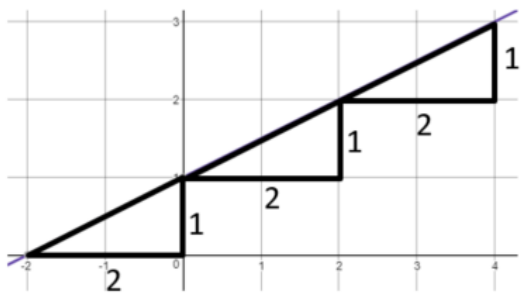
....

....

5. Complete the 'four representations' tables below.
   Note: Use the concept corner above to help.

a)

$$y = ...x + 1$$

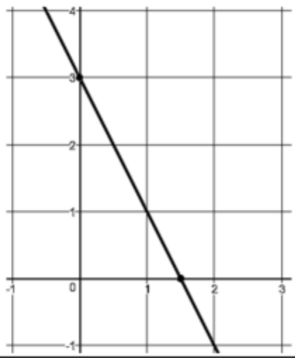


A straight line,
with gradient ....,
and y-intercept
(0,......).

x	0	1	2	3
y				

b)

$y = \ldots\ldots x + 3$

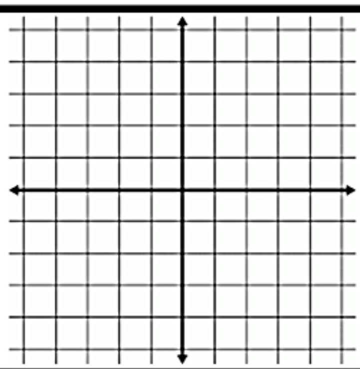


A straight line,
with gradient ....,
and y-intercept
(0,......).

x	0	1	2	3
y				

c) Make up your own.

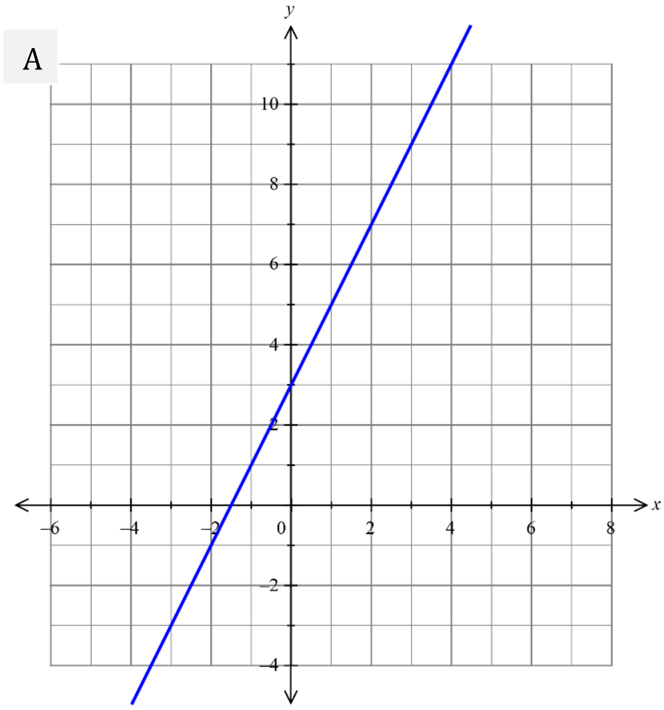
$y = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

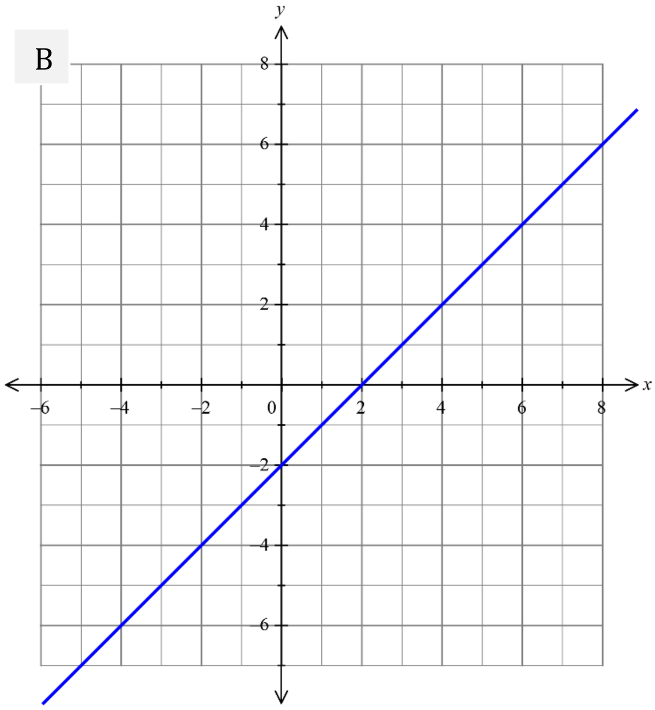


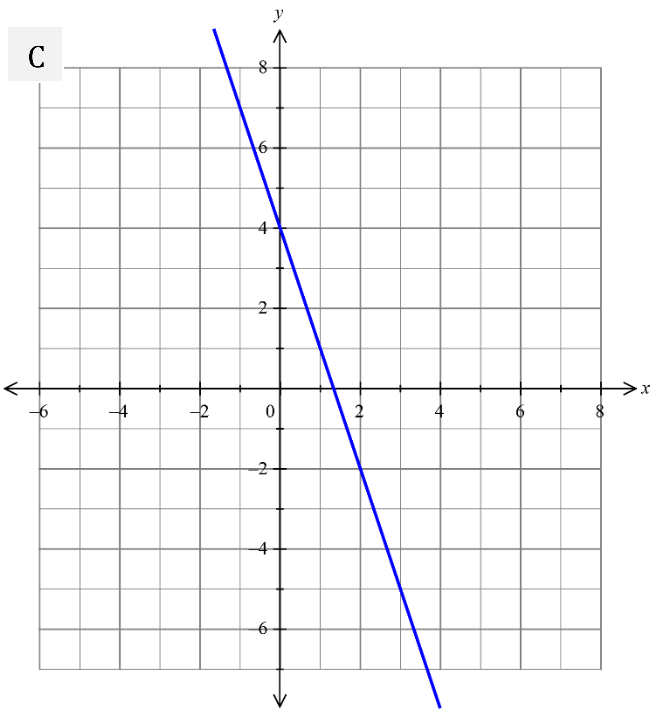
A straight line,
with gradient ....,
and y-intercept
(0,......).

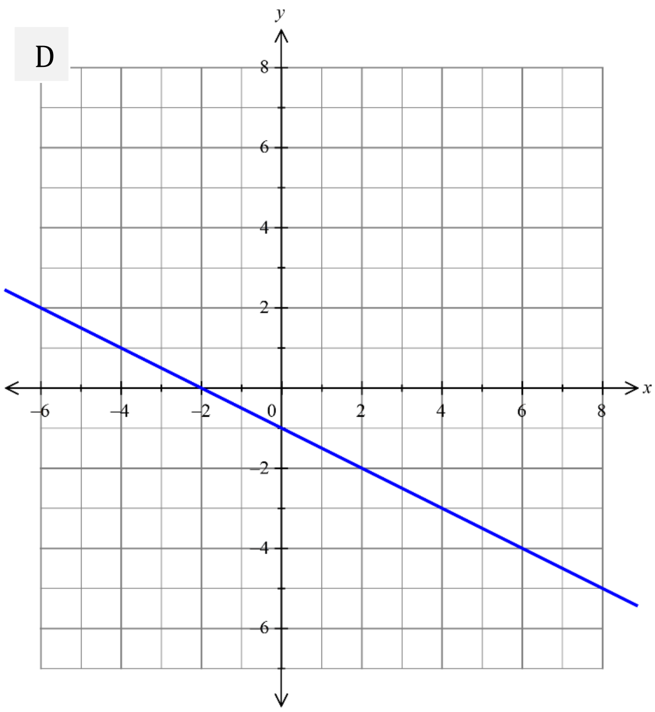
x	0	1	2	3
y				

6. Write down an equation for each line below.



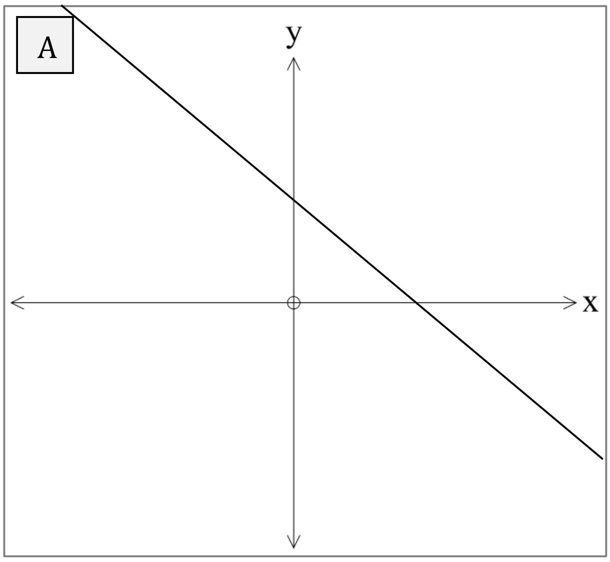


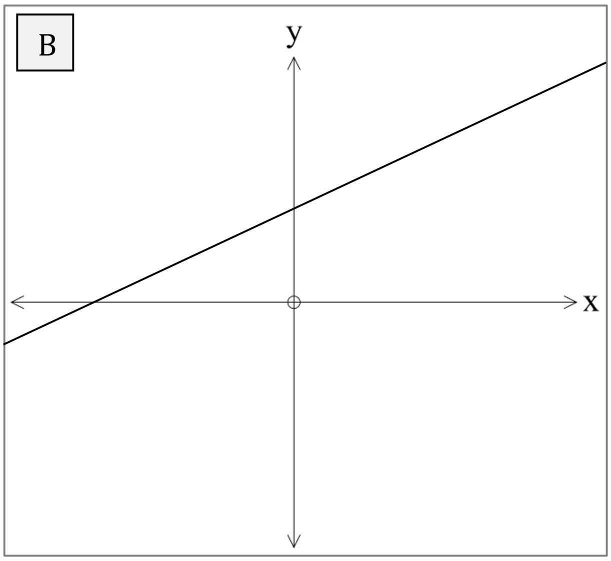


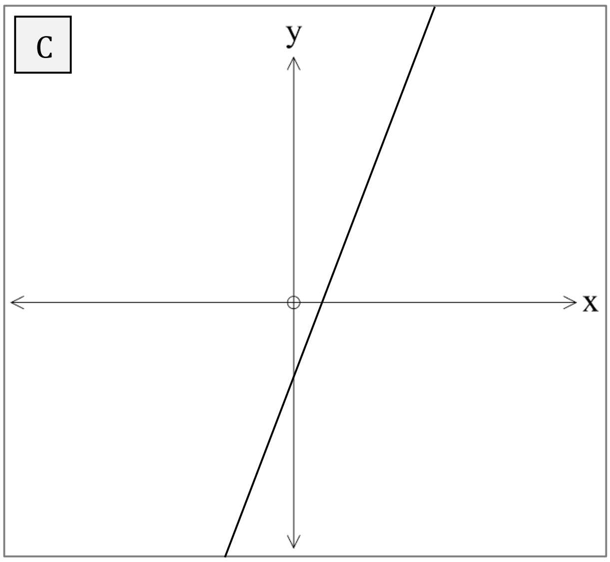


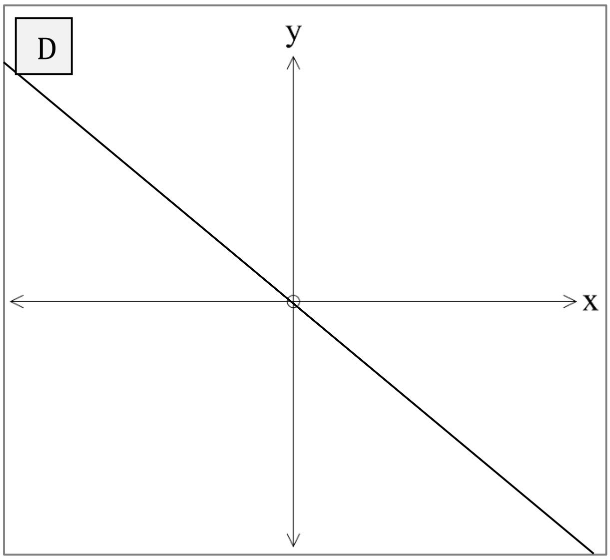
7. Match the equation of the line to the graphs below

$y = -5x + 7$	$y = -2x$	$y = 0.5x + 2$	$y = 3x - 7$









Why do graphs A and D appear to be parallel, yet their equations have different gradients?

……

2.5 Solving problems with straight lines

1. Match up the pairs of lines with the same gradient.

$y = 4x + 7$	$y = 1 - 0.25\,x$
$y = -4x + 3$	$y = 2x$
$y = -\frac{1}{4}x - 9$	$y = 4x - 3.5$
$y = 2x + 7$	$y = -4x$

2. Write an equation of a line which is parallel to the line $y = 2x - 4.5$.

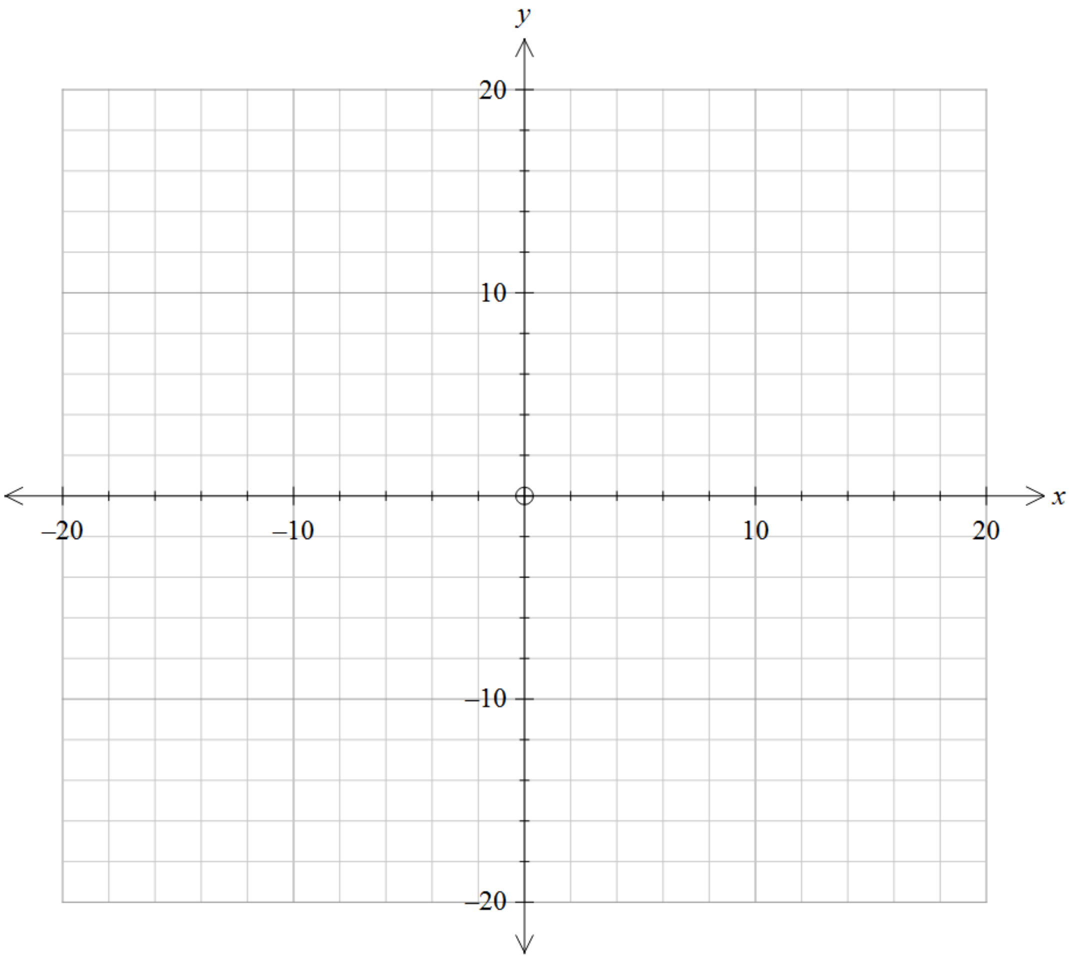
3. Sofia draws two line segments. She draws AB between points $A(1, 1)$ and $B(4, 2)$, and CD between points $C(1, -2)$ and $D(-2, -3)$.
   Show that the line segment AB is parallel to the line segment CD.

4. Write down an equation of the line that passes through the point $(0, 7)$ and has a gradient of 2.

5. Find an equation of the line parallel to the line $y = 2x - 6$ and which passes through the point $(2, -1)$.

6. Find an equation of the line passing through the points $(0, 7)$ and $(-5, 17)$.

7. Find an equation of the line that passes through the points $(4, 4)$ and $(2, -6)$.
You could use the axes below to help.



Reflections

This space is for you to write your reflections on the whole unit on linear graphs.

You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

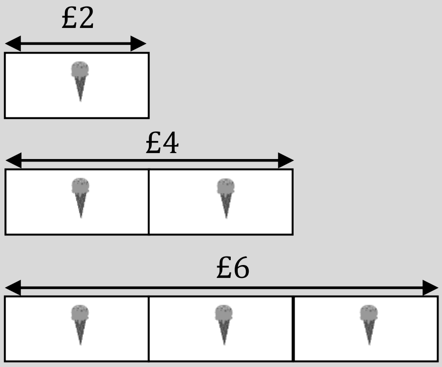
Chapter 3: Direct and inverse proportion

3.1 Direct proportion

Concept Corner

Two quantities are said to be directly proportional if one quantity is always a constant multiple of the other.

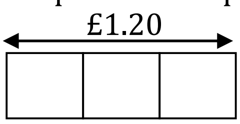
For example:
Ice-creams are sold at a cost of £2 each.
The bar models to the right show how the total cost changes as the quantity increases.



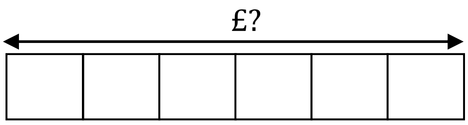
Use the information to complete the table.

Number of ice-creams	0	1	2	3	4	10
Total cost £	0	2				

1. **Three** packs of crisps cost £1.20.

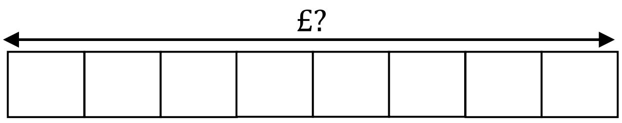


a) How much will **six** packs of crisps cost?



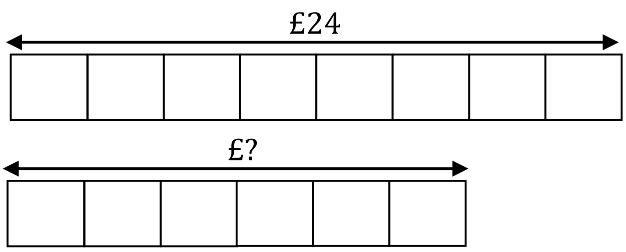
…………………………

b) How much will **eight** packs of crisp cost?



…………………………

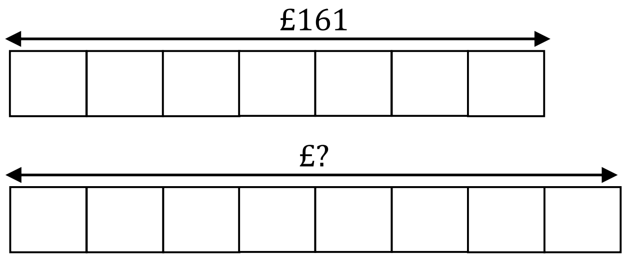
2. **Eight** small cheese pizzas cost £24.
How much will **six** small cheese pizzas cost?



…………………………

3. A meal in a restaurant costs a fixed price for each person.
   For **seven** people the total cost is £161.

   What is the total cost for **eight** people?



…………………………

4. **Seven** chocolate bars cost £3.15 altogether.

   a) How much will **three** chocolate bars cost?

…………………………

   b) How much will **ten** chocolate bars cost?

…………………………

5. **1 litre** of fruit punch contains 200 ml of apple juice and is enough for 12 people.
   a) How much apple juice is there in **1.5 litres** of fruit juice?

…………………………

   a) Harry used **2 litres** of apple juice for the same recipe.
   How many people was this enough for?

…………………………

6. Here are the ingredients to make a fish pie for **two** people.
   I want to make a fish pie for **five** people.

   How many grams of fish should I use?

Fish pie for two	
250 g	fish
350g	potatoes
20g	butter

…………………………

7. Karen bought 12 individual packs of crisps priced at 70 pence each.

   How much money could she have saved if she had bought multi-pack bags of 6, each at a cost of £2.79?

…………………………

8. The table below shows the ingredients needed to make 12 doughnuts.

   Complete the table to work out how much of each ingredient you need if you scale the recipe to make 6, 24 or 15 doughnuts.

Ingredients	12 doughnuts	6 doughnuts	24 doughnuts	15 doughnuts
flour	200 g			
margarine	40 g			
milk	60 ml			
sugar	50 g			
eggs	2			$2\frac{1}{2}$

9. This is a recipe for fruit punch for **six people**:

Fruit punch	
300 g	chopped pineapple
1500 ml	orange juice
750 ml	apple juice

Jake made fruit punch for **ten people**.
How many millilitres of orange juice did he use?

…………………………

10. Philippa went on holiday to Sweden.

The exchange rate is £1 to 1.2 euros (€).
She exchanges £550 into euros.

a) How many euros should she receive?

……………………………………………

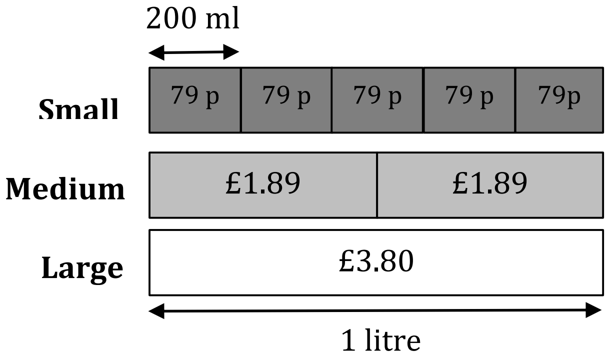
Philippa returned home with 78 euros left.
The new exchange rate was £1 to 1.25 euros.

b) How many pounds did she get back?

……………………………………………

11. Anne is buying some orange juice.
The following sizes are available:

Small	200 ml	79p
Medium	500 ml	£1.89
Large	1 litre	£3.80

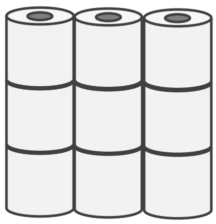


Which is the best value for money? Justify your answer, using the bar model to help you.

……………………………………………

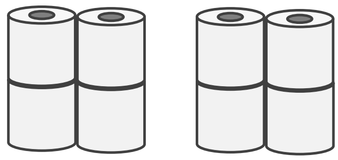
12. Jason wants to buy some toilet roll. He has the following two options:

Option A



9 rolls for £4.49

Option B



2 packs of 4 rolls for £3.75

Which option is the best value for money?
Show your working.

……………………………………………

13. The distance travelled by a car (assuming the car is travelling at a steady speed) is directly proportional to the time taken.
A car travels 125 kilometres in $2\frac{1}{2}$ hours.

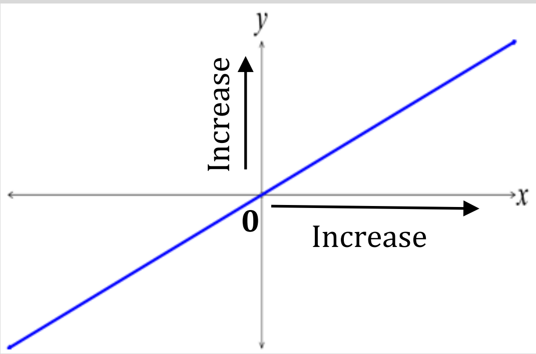
How far will the car travel in 6 hours?

…………………………………………

Concept corner

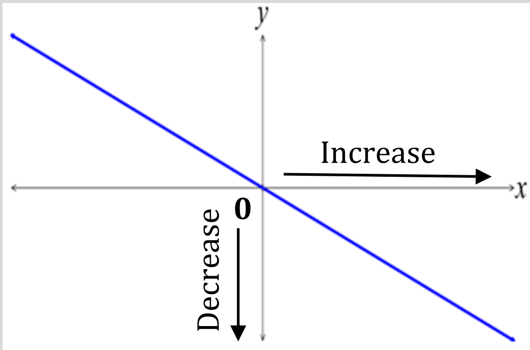
Direct proportion can be represented graphically with the line $\boldsymbol{y = ax}$ passing **through the origin.**

1. when $\boldsymbol{a > 0}$



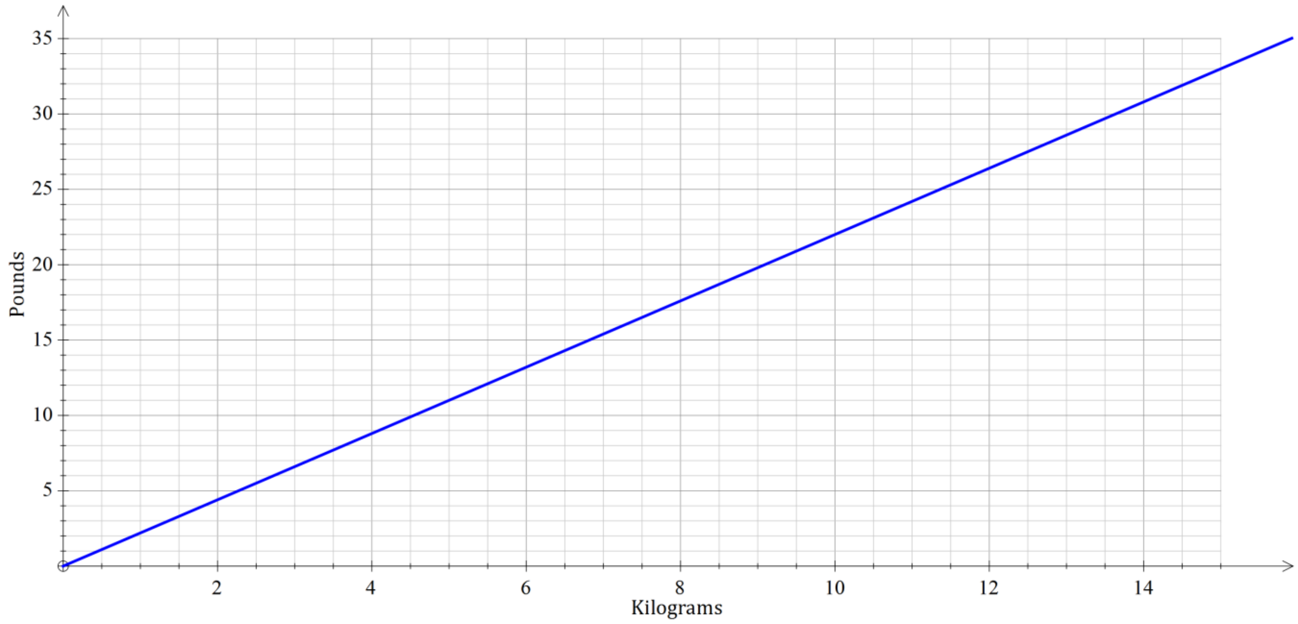
The gradient is positive.

2. when $\boldsymbol{a < 0}$



The gradient is negative.

14. The graph below shows the approximate conversion between pounds (lb) and kilograms (kg).



a) Use the graph to convert 10 kilograms to pounds.

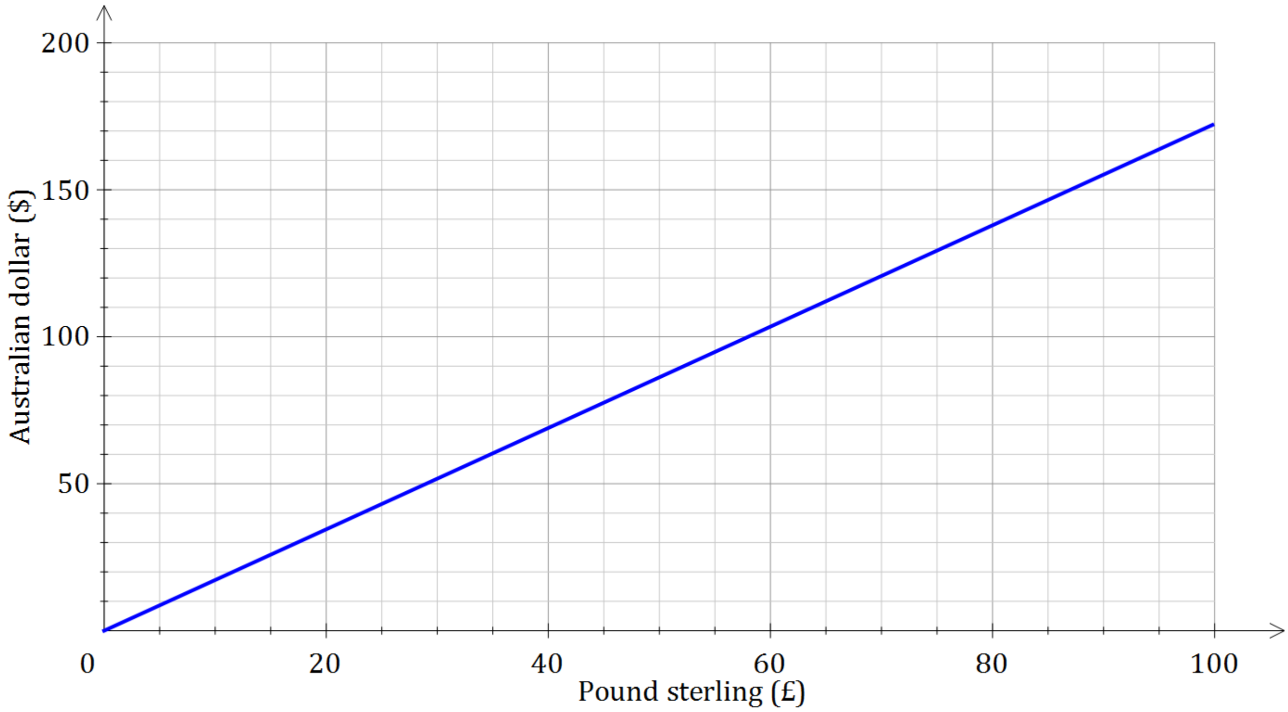
b) Use the graph to convert 10 pounds to kilograms.

c) What is the difference between 10 kilograms and 10 pounds?
Give your answer in pounds and kilograms.

d) What is the difference between 15 kilograms and 15 pounds?
Give your answer in pounds and kilograms.

e) Sofia weighs 75kg. She says that she is heavier than Jack who weighs 154 pounds.
Is she correct?

15. Here is a conversion graph between pound sterling (£) and the Australian dollar ($).



a) Change £40 to Australian dollars.

b) Change 100 Australian dollars to pounds sterling.

c) What is the difference between £100 and $100?
Give your answer in pound sterling and Australian dollars.

d) Caitlin converted £560 to dollars.
How many dollars did she receive?

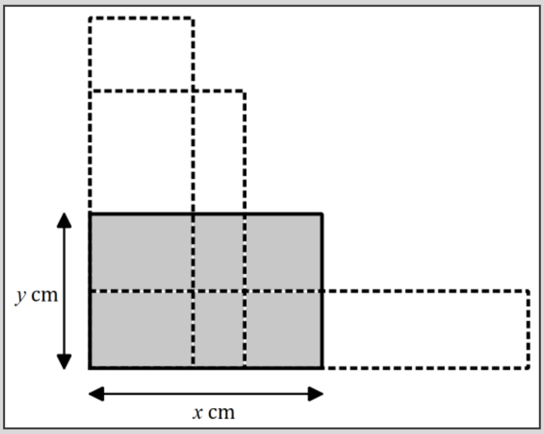
3.2 Inverse proportion

Concept Corner

Consider a rectangle of area 18 cm^2 with dimensions x cm and y cm.

This relationship is shown in the table below.

x	1	2	3	4	5	6	...
y	18	9	6	4.5	3.6	3	



As the length of the horizontal side becomes 2 times, 3 times and 4 times as long, the length of the vertical side changes by a factor of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ accordingly.

If the length of the horizontal side is represented as x cm and the length of the vertical side is represented as y cm, the relationship between x and y can be represented with the following equation.

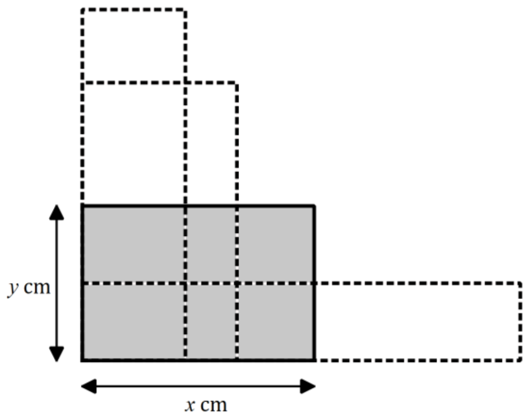
$$xy = 18$$

Two variable quantities are said to be **inversely proportional** if the **product of the two quantities is constant.**

1. Decide whether the scenarios below represent direct or inverse proportion:

Scenario	Direct	Inverse
The cost of apples and the quantity of apples.		
The speed travelled and the time taken.		
The time it takes to build a house compared with the number of builders all working at the same rate.		
The circumference of a circle and its diameter.		

2. A rectangle of area 24 cm^2 has dimensions x cm and y cm.



Complete the table to show relationship between the two lengths.

x	1	2	3	4	5	6	7	8	9	10	11	12
y	24				$\frac{24}{5}$							

3. On a farm many years ago the water tanks were filled using a bucket from a well.
The table shows the numbers of buckets of different capacities needed to fill a tank of capacity 1200 pints.

a) Complete the table:

Capacity of bucket (pints)	1	2	3	4	5	6	8	10
Number of buckets						200		

b) Write a formula to connect the capacity of the tank, the capacity of a bucket ($\boldsymbol{b}$) and the number of buckets ($\boldsymbol{n}$).

4. The Beta construction company is building an extension.
Three builders take 15 days to complete a project.

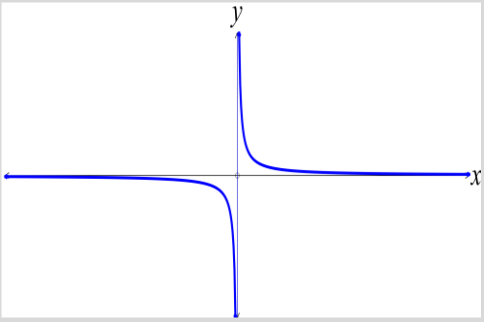
a) How many days would it take one builder?

b) How long would the project take to complete if there were six builders?

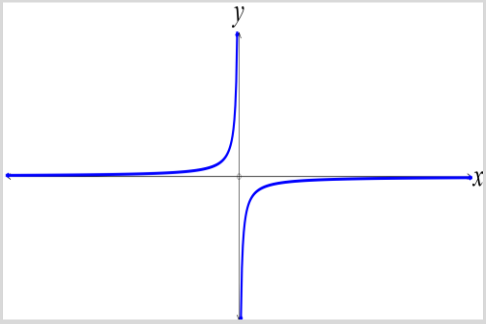
Concept corner

Inverse proportion can be represented graphically using $y = \frac{a}{x}$. This is a curved line.

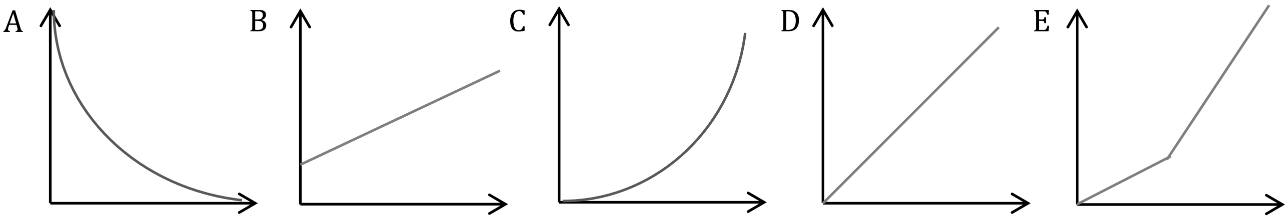
1. when $a > 0$



2. when $a < 0$



5. The graphs below show data collected from some events.



Identify whether each of the graphs represent direct proportion, inverse proportion or neither.

Graph	Direct	Inverse	Neither
A			
B			
C			
D			
E			

6. Use the key words below to compare and contrast the graphs for direct and inverse proportion (see the concept corners for hints).

linear   curved   origin   constant

………

………

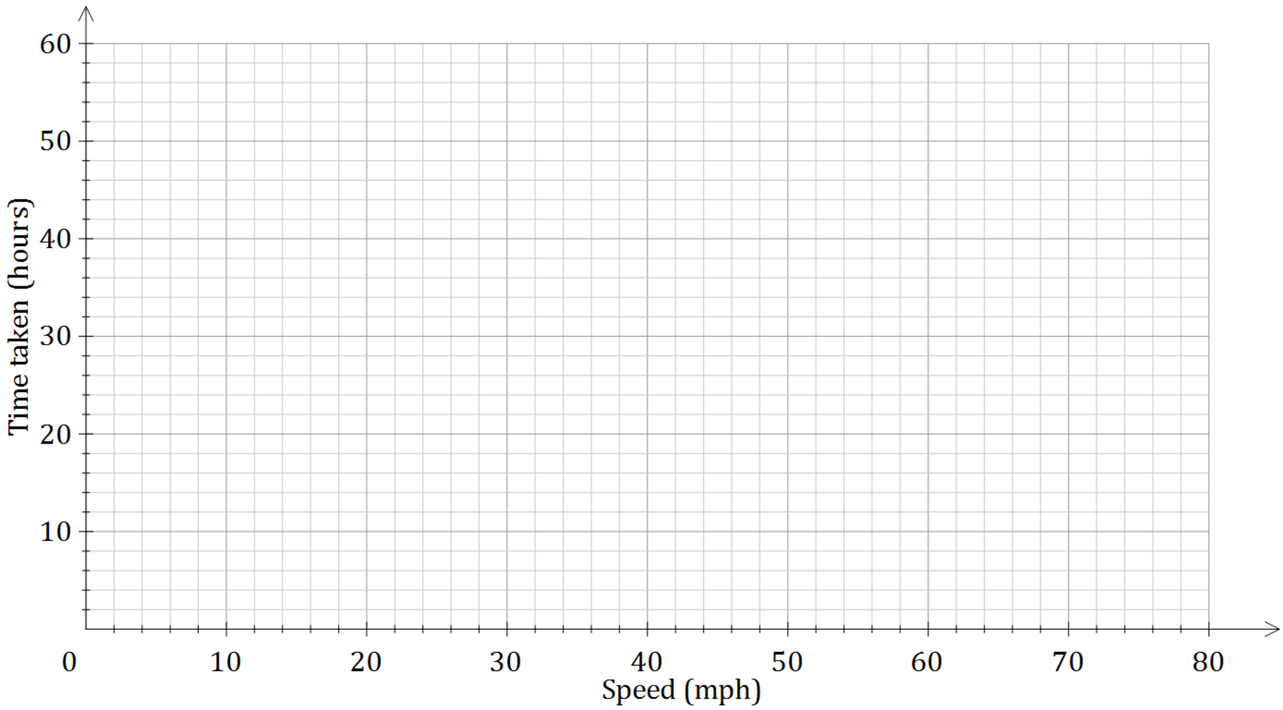
………

7. Bill and Ben are planning a 240-mile journey from London to Paris. They want to know how long it will take if they travel at different speeds.

a) Complete the table below to show the time taken.

Speed (mph)	5	10	20	30	40	48	60	80
Time taken (hours)								

b) Plot this information on the graph below.



3.3 Mixed problems

1. This is the recipe for 12 pancakes:

12 Pancakes

100 g plain flour

2 large eggs

300 ml milk

3 lemons

30 g caster sugar

Ashley has the following ingredients in his cupboard.

250 g plain flour
6 large eggs
1 litre milk
8 lemons
100 g caster sugar

What is the maximum number of pancakes that he can make?

You may find the bar models below useful.

Flour:	100 g	100 g	50 g		(30 pancakes)
Eggs:	2 eggs	2 eggs	2 eggs		(36 pancakes)
Milk:	300 ml	……………	……………	…	(…… pancakes)
Lemons:					
Sugar:					

……………………………………………

2. The time in days, T, it takes to build a house is inversely proportional to the number of builders, N, working on the house.

   8 builders take 25 days to build a house.

   a) How many days would 4 builders take?

   b) How many days would 10 builders take?

3.

a) A table of values for a and b is shown below.
a is directly proportional to b.

Complete the table.

$\boldsymbol{a}$	2	3	7	10	20	50	100
$\boldsymbol{b}$		15		50			
$\boldsymbol{b \div a}$		5					

b) A table of values for m and n is shown below.
m is inversely proportional to n.

Complete the table.

$\boldsymbol{m}$	2	4	5	10	20	40
$\boldsymbol{n}$	250			50		
$\boldsymbol{m \times n}$	500					

c) Explain what you notice about the values of $\boldsymbol{b \div a}$ and $\boldsymbol{m \times n}$.

4. The amount of water used when taking a shower (assuming a steady flow) is directly proportional to the time it takes to shower.
Ali uses 65.1 litres in 8.2 minutes.

How much water would he use in half an hour?
Round your answer to sensible degree of accuracy.

5. If Jo takes 3 hours to paint a room and Sam takes 5 hours, how long would they take if they painted a room together?

Reflections

This space is for you to write your reflections on the whole unit direct and inverse proportion.

You may wish to write about:

- Things you’ve learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

Chapter 4: Standard form

4.1 Large numbers

1. Write the numbers below in words:

   a) 5 600 003 ……………………………………………………………………………………………………

   ……

   b) 91 345 000 …………………………………………………………………………………………………

   ……

   c) 9 000 000 999 ……………………………………………………………………………………………

   ……

2. Write the numbers below in figures:

   a) Eight hundred and four million, two hundred and three.

   ……

   b) Two and a half billion.

   ……

   c) Five billion, three hundred and fifty-five million, four hundred and eighty.

   ……

3. Round these populations to two significant figures.

   a) Athens 4 173 542 ……………………………………………………………

   b) Rome 2 627 336 ……………………………………………………………

   c) London 8 673 713 ……………………………………………………………

   d) Paris 2 244 712 ……………………………………………………………

   e) Finland 5 439 258 ……………………………………………………………

4. Complete the table below:

Number	Power of 10	Written
10	10^1	Ten
100	10^2	
1000		
10 000		
100 000	10^5	
1 000 000		
1 000 000 000		

Concept Corner

Standard form is used to write very large or very small numbers.

n is an integer

$$a \times 10^n$$

$1 \leq a < 10$

For example:

55 000 000 can also be written as 5.5×10^7.

1. Identify the missing values below:

a) 900 = $9 \times 10^?$ ……………………………….

b) 90 000 = $9 \times 10^?$ ……………………………….

c) 84 = $8.4 \times 10^?$ ……………………………….

d) 8 400 000 = $8.4 \times 10^?$ ……………………………….

e) 8 400 000 000 = $8.4 \times 10^?$ ……………………………….

2. Complete the table below:

Number	**Standard form**
6000	
9 000 000 000	
	5.1×10^2
43 000 000	
	1.71×10^4
579 100 000 000	

3. Complete these sentences:

   a) The number 4.6×10^5 is ……………………….. times bigger than 4.6×10^4

   b) The number 4.6×10^8 is ……………………….. times smaller than 4.6×10^{10}

   c) The number 4.6×10^7 is ……………………….. times bigger than 4.6×10^3

   d) The number 4.6×10^4 is ……………………….. times bigger than 2.3×10^3

   e) The number 4.6×10^2 is ……………………….. times smaller than 1.15×10^6

4. Write these numbers in ascending order.

   2.9×10^3 29 000 29×10^1 0.29×10^6 29×10^0

………

5. Using the information below, calculate the mass of the asteroids A to E, leaving your answers in **standard form**.

- Asteroid A is twice as big as asteroid C.
- Asteroid D has a mass of 5.1×10^3 kg.
- Asteroid C is a third of the size of Asteroid D.
- Asteroid B is 1.1×10^3 kg heavier than Asteroid C.
- The masses of Asteroid B and E are in a 2 : 3 ratio to each other.

Asteroid	A	B	C	D	E
Mass (kg)					

4.2 Small numbers

Concept Corner

Complete these statements by following the pattern:

10^3 = 1000 ÷ 10

…… = 100 ÷ 10

10^1 = ……………………….

10^0 = ……………………….

10^{-1} = ……………………….

…… = 0.01

10^{-3} = 0.001

1. Shade, or use a key to show the matching terms in the grid below. The first one has been completed for you.

10^2	0.01	Ones	$\frac{1}{100}$	10^{-1}	Thousandths
$\frac{1}{1}$	Tenths	Tens	100	0.001	10^0
$\frac{1}{10}$	10^1	1	10^{-2}	Hundreds	$\frac{1}{1000}$
Hundredths	0.1	10	$\frac{100}{1}$	$\frac{10}{1}$	10^{-3}

2. Complete the table:

Product	**Multiplication**	**Standard form**
0.3	3×0.1	3×10^{-1}
	5×0.01	5×10^{-2}
0.007		
	4×0.0001	
		2×10^{-6}
0.000 000 006		

3. Complete these sentences:

a) The number 8.4×10^{-2} is .... times bigger than 8.4×10^{-3}

b) The number 8.4×10^{-2} is .... times bigger than 8.4×10^{-4}

c) The number 8.4×10^{-5} is .... times bigger than 8.4×10^{-8}

d) The number 8.4×10^{-3} is .... times smaller than 8.4×10^{3}

e) The number 8.4×10^{-3} is .... times smaller than 4.2×10^{-1}

4. Draw lines to match the cards of equivalent value.

5×10^{-3}	0.005 05	$\frac{55}{1\,000\,000}$
5.5×10^{-4}	0.005	$\frac{505}{1\,000\,000}$
5.5×10^{-5}	0.000 505	$\frac{5}{1000}$
5.05×10^{-4}	0.000 055	$\frac{55}{100\,000}$
5.05×10^{-3}	0.000 55	$\frac{505}{100\,000}$

5. Write these numbers in ascending order.

2.9×10^{0} 0. 000 29 2.9×10^{-3} 0.29×10^{-1} 2.9×10^{2}

6. The atomic radii of elements are often measured in picometers (1 pm = 1×10^{-12} m). Using the information below, calculate the radii of the following elements, giving the lengths of each in **standard form** in metres.

- The radius of Phosphorus (P) is four times as big as Hydrogen.
- The radius of Aluminium (Al) is 25% larger than Phosphorus.
- The radius of Hydrogen (H) is 25 picometers.
- Mercury (Hg) is 2.5×10^{-11} metres larger than Aluminium.
- The radii of Phosphorus and Scandium (Sc) are in the ratio 5 : 8.

Element	Phosphorus	Hydrogen	Aluminium	Mercury	Scandium
Radius (m)					

4.3 Mixed standard form questions

1. Convert the following numbers into standard form correctly.

   a) 43.8×10^{6} ……………………………………………………………………………………..

   b) 438×10^{4} ……………………………………………………………………………………..

   c) 438×10^{-5} ……………………………………………………………………………………..

   d) 43.8×10^{-4} ……………………………………………………………………………………..

   e) 0.438×10^{-3} ……………………………………………………………………………………..

2. The radius of the Earth is 6370 kilometres. Convert this distance to metres, centimetres and millimetres, writing your answers in standard form.

Metres: ……………………….. Centimetres: ……………………….. Millimetres: ………………………..

3. Calculate the number of seconds in one century. Write your answer in standard form.

………………………..

4. The surface areas of three planets Mercury, Venus and Earth are 7.48×10^{7} km^2, 460 000 000 km^2 and 510 million km^2 respectively.
   Which has the largest surface area?

………………………..

4.4 Scales

1. The pairs of ratios written in each box are equivalent. Use the numbers in the cloud below to fill in the missing values.

275 25.5 0.25 4 10 500 6

1 cm : 3000 cm …………cm : 12 000 cm	1 : 80 ………… cm : 480 cm	1 cm : 400 cm ………… km : 100 km
2 : 5 110 cm : ………… cm	1 inch : 3000 cm 3.5 inches : ………… cm	2 inches : 3 feet 17 inch : ………… feet

2. On a diagram, the length of an aeroplane's wingspan is 22 cm.
   The diagram is drawn to the scale 1 cm : 3 m.
   What is the real length of the wingspan?

………

3. The length of Stacey's garden fence is 18 metres. On the plan drawings it was 3 cm. What was the scale of her plan drawing?

………

4. Will wants to draw some scale drawings. Match up what he is trying to draw with an appropriate scale.

A house	1 cm : 1 km
A bedroom	$1 : 2.5 \times 10^7$
A city	1 cm : 10^9 km
The Solar System	2 cm : 1 m
The Earth	1 : 6

5. Two maps of London have different scales. Map A has a scale of 1 : 10 000 and Map B has a scale of 2 : 15 000.

   a) On which map would the London Eye image be larger?

……..

   b) Explain your answer.

……..

……..

Reflections

This space is for you to write your reflections on the whole unit on standard form.

You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

Chapter 5: Sequences

5.1 n^{th} term rule

1. Fill in the missing numbers and write the term-to-term rule for each sequence.

a) $\boxed{8} \rightarrow \boxed{12} \rightarrow \boxed{16} \rightarrow \boxed{} \rightarrow \boxed{}$

Term-to-term rule:

b) $\boxed{5} \rightarrow \boxed{} \rightarrow \boxed{13} \rightarrow \boxed{17} \rightarrow \boxed{}$

Term-to-term rule:

c) $\boxed{21} \rightarrow \boxed{17} \rightarrow \boxed{13} \rightarrow \boxed{} \rightarrow \boxed{}$

Term-to-term rule:

d) $\boxed{1} \rightarrow \boxed{} \rightarrow \boxed{} \rightarrow \boxed{13} \rightarrow \boxed{}$

Term-to-term rule:

e) $\boxed{3} \rightarrow \boxed{} \rightarrow \boxed{} \rightarrow \boxed{-9} \rightarrow \boxed{}$

Term-to-term rule:

f) $\boxed{-7} \rightarrow \boxed{} \rightarrow \boxed{} \rightarrow \boxed{} \rightarrow \boxed{-19}$

Term-to-term rule:

2. Write down the first 5 terms in the sequence $4n - 1$.

3. George thinks that the n^{th} term rule for the sequence 7, 11, 15, 19, 23 is $3n + 4$.
He is wrong.
Explain the mistake that he has made.

………

4. True or false:
    a) 35 will be in the linear sequence 3, 8, 13, 18, 23, … True/False

    b) There will be a multiple of five in the arithmetic sequence 4, 13, 22, 31, ...

    True/False

    c) The n^{th} term rule for the sequence 7, 14, 21, … is $n + 7$ True/False

    d) The n^{th} term rule for the sequence 7, 13, 19, … is $6n + 1$ True/False

5. Circle the n^{th} term expressions below where 45 would be the 6^{th} term in the sequence.

$n^2 - 23$ $7n + 3$ $10n - 15$ $2.5n + 30$ $15 - 5n$

6. Match each sequence to the correct n^{th} term rule:

$3n + 5$	$-2, 1, 4, 7, \ldots$
$5n + 3$	$2, -1, -4, -7, \ldots$
$3n - 5$	$8, 11, 14, 17\ \ldots$
$5n - 3$	$8, 13, 18, 23, \ldots$
$5 - 3n$	$2, 7, 12, 17, \ldots$

7. Work out the n^{th} term rule for the following arithmetic sequences:

a) 7, 12, 17, 22, …

b) 12, 18, 24, 30, …

c) −3, 1, 5, 9, …

d) −5, −10, −15, −20, …

e) 7, −3, −13, −23, …

f) 4.7, 5, 5.3, 5.6, …

g) $\frac{1}{5}, \frac{4}{5}, 1\frac{2}{5}, 2, \ldots$

h) $\frac{1}{2}, \frac{5}{6}, 1\frac{1}{6}, 1\frac{1}{2}, \ldots$

i) $1.4 \times 10^4, 1.6 \times 10^4, 1.8 \times 10^4, \ldots$

8. Circle the odd one out in each set of four below:

$2n + 4$	$2(n + 2)$	$6, 8, 10, 12, \ldots$	$6 + 2(n + 1)$
$6n - 15$	$3(2n - 5)$	$21, 27, 33, 39, \ldots$	$-9 + 6(n - 1)$
$40 - 16n$	$-8(2n - 5)$	$24, 8, -8, -24, \ldots$	$40 - 16(n - 1)$

9. Jake writes out the first ten terms of a sequence:

3, 5, 7, 9, 11, 13, 15, 17, 19, 21…

He thinks the 100th term would therefore be 2100.

a) What mistake has he made?

……

……

b) What is the 100th term?

…………………………..

10. Fill in the gaps in the following arithmetic sequences and work out the n^{th} term rule:

a) ………, ………, ………, 19, 26 n^{th} term: …………………………..

b) ………,………, 10, ………, 30 n^{th} term: …………………………..

c) −7, ………, ………, 2, ……… n^{th} term: …………………………..

d) −16, ………, ………, ………, 4 n^{th} term: …………………………..

11. Find the n^{th} term rule for the arithmetic sequence where the 1st term is 9 and 5th term is 25.

…………………………..

12. Find the n^{th} term rule for the arithmetic sequence where the 4th term is 33 and the 7th term is 21.

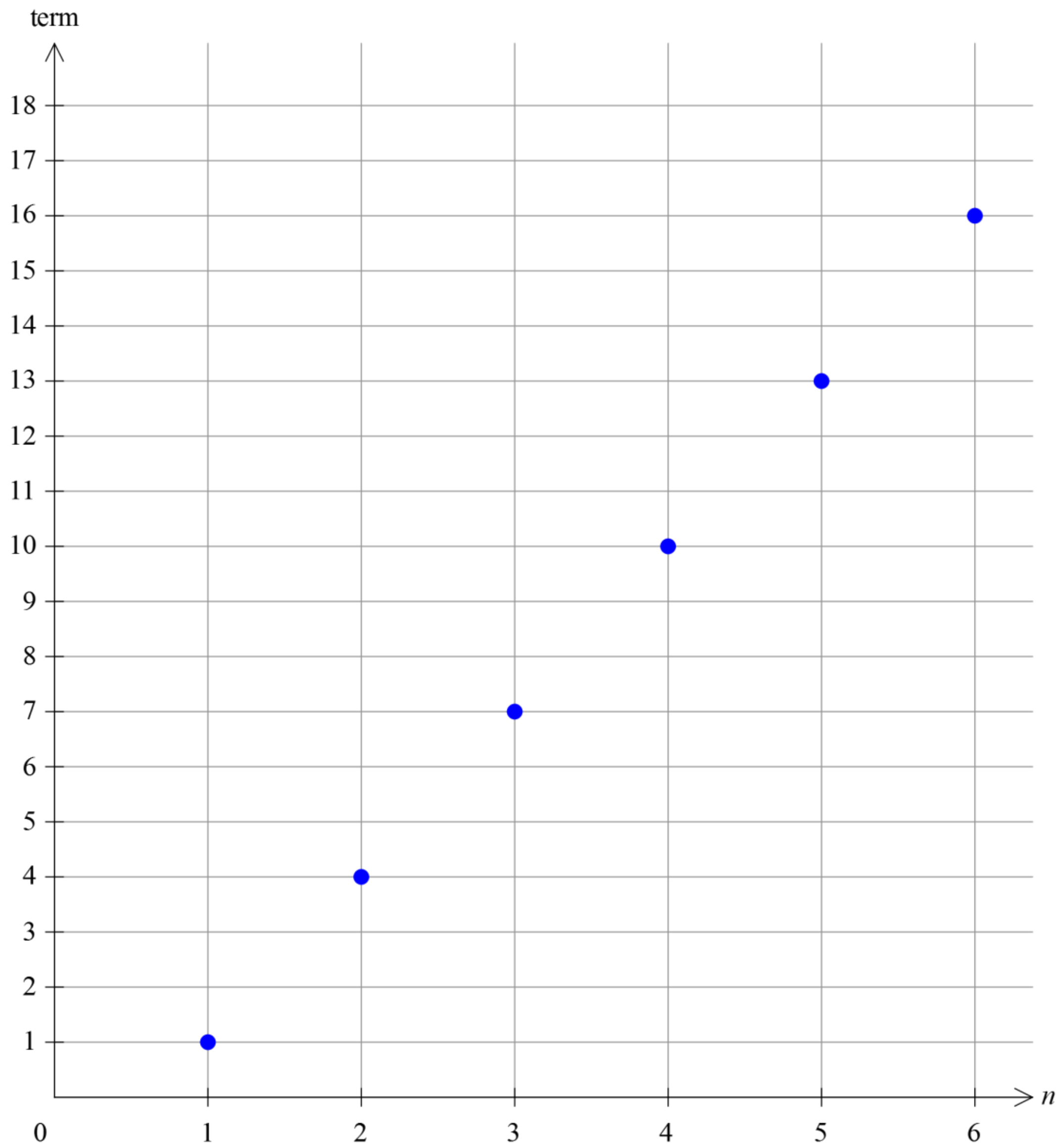
…………………………..

13. The first World Cup took place in 1930 and takes place every four years after that.
No World Cup took place in 1942 and 1946.
In what year should the 100th World Cup take place?

…………………………..

5.2 Linear sequences connect to linear graphs

1. The graph below represents the sequence $3n - 2$.



a) Explain how the graph can be used to show that 6 is *not* a term in the sequence.

....

....

b) Give one reason why $3n - 2$ and $y = 3x - 2$ are similar, and one reason why they are different.

....

....

2. $$3n - 4$$

a) Sequence or graph? ....

b) What does n represent? ....

c) What is the first term? ....

d) What are the first 3 terms? ....

e) What is the term-to term rule? ....

3. $$y = 3x - 4$$

a) Sequence or graph? ....

b) What do x and y represent? ....

c) What is the y value when $x = 0$? ....

d) What are the y values when $x = 1, 2$ and 3? ....

e) What is the gradient of the line? ....

4. $$4(2 - n)$$

a) Sequence or graph? ....

b) What does n represent? ....

c) What is the first term? ....

d) What are the first 3 terms? ....

e) What is the term-to term rule? ....

5.3 Linear and non-linear sequences

1. a) Generate the first five terms of the following sequences:

   i. $3n - 5$ ....

   ii. $15 - 6n$ ....

   iii. $n^2 - 5$ ....

   iv. $\frac{n^3+3}{2}$ ....

   v. 2^n ....

   b) What do you notice about the difference between each term in the sequences above?

   ....

   ....

   c) Sort the sequences into two groups based on your answer to part b.

2. For each of the sequences in the table, decide if the sequence is linear or non-linear.

n^{th} term	Linear	Non-linear
$n + 4$		
$3^2 - 2n$		
$n^3 - 2n$		
$\frac{1-n}{n+1}$		
$3(1 + 2n)$		

1. Match the terms to the correct n^{th} term rule. Terms may be in more than one sequence.

$2n+3$	28
$3n^2$	27
$3n+2$	29
n^2+3	

2. a) Complete the next three lines in the following number pattern

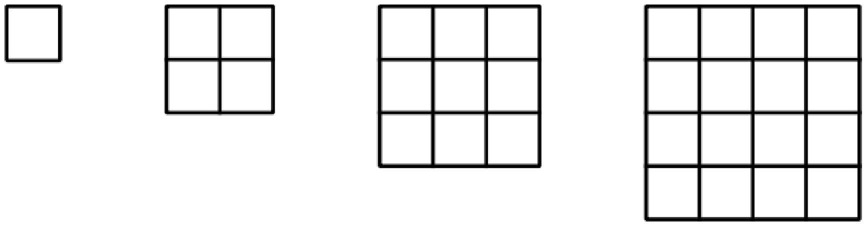
1 + 3 = 4
1 + 3 + 5 = 9
1 + 3 + 5 + 7 = 16
…………………… = ………..
…………………… = ………..
…………………… = ………..

b) What is the special name given to the numbers 4, 9, 16?

3. Match up each sequence to the correct description:

Square numbers	1, 1, 2, 3, 5, …
Cube numbers	2, 4, 8, 16, 32, …
Triangular numbers	1, 4, 9, 16, 25, …
Powers of 2	1, 3, 6, 10, 15, …
Fibonacci sequence	1, 8, 27, 64, 125, …

4. Preeti uses the following diagrams to represent part of a sequence.



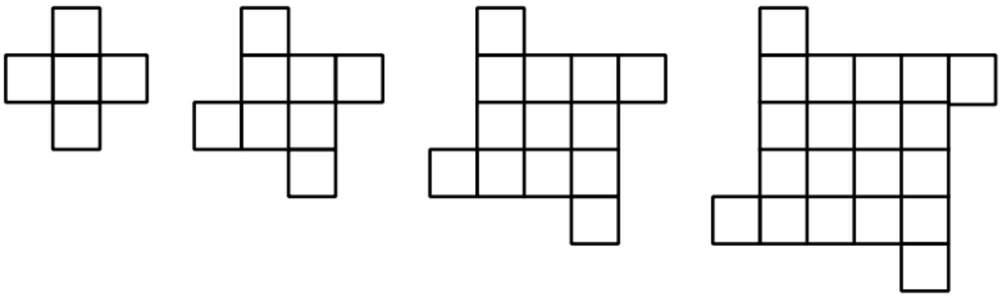
a) What is the name of the special types of number that these diagrams represent?

……………………….……………………….....

b) What is the n^{th} term rule for this sequence?

………………………....

c) Preeti adapts her sequence to look like this:



What is the n^{th} term rule of the new sequence?

………………………....

d) Will 99 be a term in the new sequence? Justify your answer.

....

....

5. Work out the n^{th} term rule for each sequence below:

   a) 1, 4, 9, 16, 25, …

   b) 2, 5, 10, 17, 26, …

   c) 2, 8, 18, 32, 50, …

   d) 4, 7, 12, 19, 28, …

   e) 4, 9, 16, 25, …

6. Decide whether the sequences below are arithmetic or geometric or neither:

Sequence	**Arithmetic**	**Geometric**	**Neither**
6, 12, 18, 24, 30 …			
5, 25, 125, 625, 3125 …			
5, 2, −1, −4, −7 …			
2, 3, 5, 7, 11 …			
−3, −6, −12, −24, −48			
−5, 0, −5, 0, −5 …			

5.4 Mixed sequences problems

1. For each sequence below, decide if the sequence is arithmetic, geometric or neither.

Sequence	Arithmetic	Geometric	Neither
5, 10, 20, 40, 80, …			
0, 3, 8, 15, 24, …			
2, 16, 54, 128, …			
10, 6, 2, -2, -6, …			
1, 2, 4, 8, 16, …			
1.6, 1.2, 0.8, 0.4, 0, …			
2, -6, 18, -54, 162, …			

2. The population of rabbits on a farm doubles every month. After 5 months there are 96 rabbits.

   i. How many rabbits were there originally?

   ………………………..

   ii. How many rabbits will there be after 12 months?

   ………………………..

3. In the first year at a new job, Joe is paid a salary of £20 000.
   His salary goes up by 4% each year.

   i. What is his salary in the 2^{nd} year?

   ………………………..

   ii. What is his salary in the 10^{th} year?

   ………………………..

   iii. In how many years will Joe have a salary of more than £35 000

   ………………………..

4. A company's profits increase by £400 every year. In 2014 the profits were £2300.

   i. How much will the profits be in the year 2019?

   ……………………….

   ii. If 2014 is year 1, can you find a formula for the n^{th} term for the profits?

   ……………………….

   iii. In which year will the profits be more than £10000?

   ……………………….

5. The rule for finding the next term in a sequence is to multiply the previous term by 2 and then add a, where a is an integer.

   The first term is 9 and the third term is 48.

   Work out the value of a.

   $a =$ ……………………….

Reflections

This space is for you to write your reflections on the whole unit on sequences.

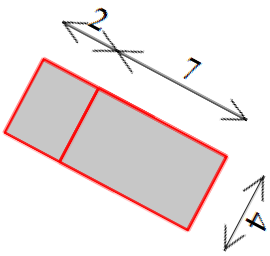
You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

Chapter 6: Expanding and factorising

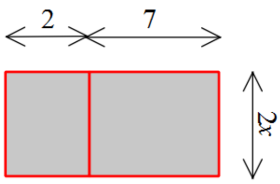
6.1 Expanding single brackets

1. Choose two expressions from the list to match each shaded area.



……………………………………

……………………………………



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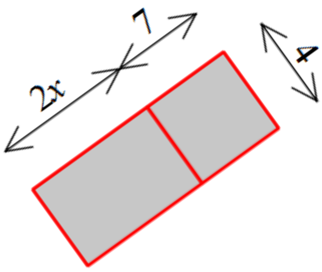
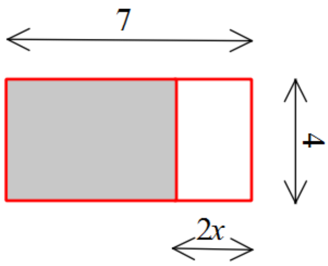


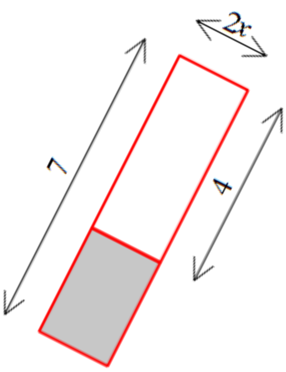
……………………………………

……………………………………



……………………………………

……………………………………



……………………………………

……………………………………

$2x(7-4)$

$8x+28$

$4(7-2x)$

$4x+14x$

$14x+28$

$4(2+7)$

$28-8x$

$4(2x+7)$

$8+28$

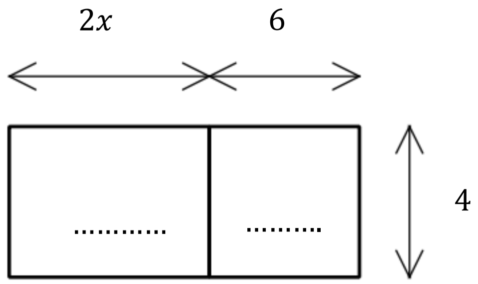
$7(2x+4)$

$2x(2+7)$

$14x-8x$

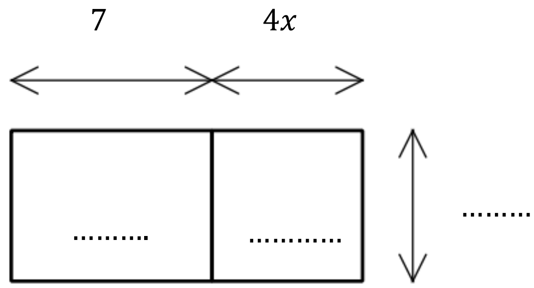
2. Expand the expressions below and label the area models appropriately.

a)



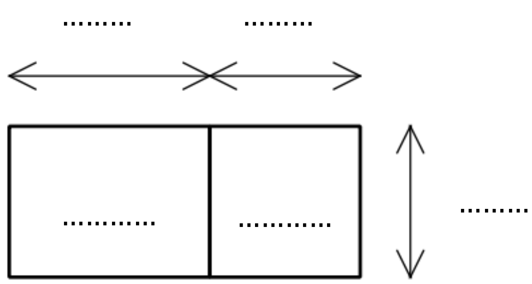
$4(2x + 6) \equiv 8x + 24$

b)



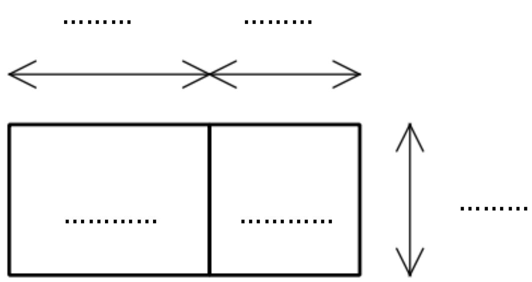
$11(7 + 4x) \equiv$ ....

c)



$9y(4x + 5) \equiv$ ....

d)



$4x\left(x + \frac{1}{2}\right) \equiv$ ....

3. Expand the following expressions:

a) $3(5x + 6)$ $\equiv$ ....

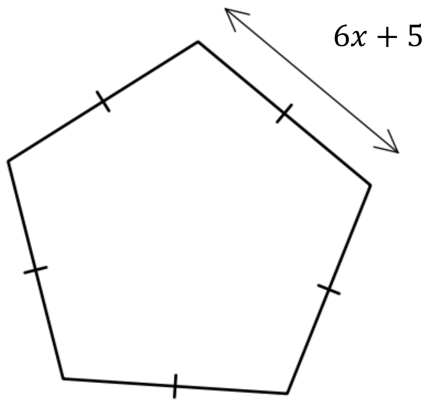
b) $11(7x - 4)$ $\equiv$ ....

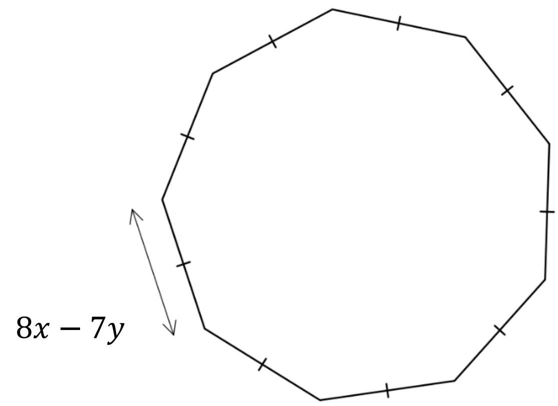
c) $2y(3x - 5)$ $\equiv$ ....

d) $4x(9 - x)$ $\equiv$ ....

4.

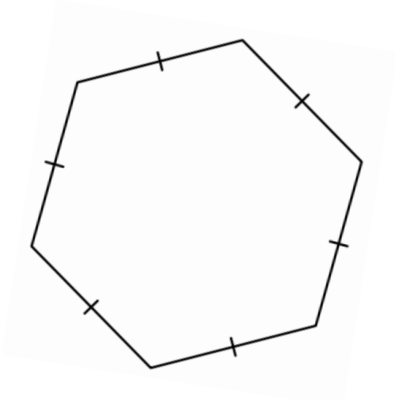
a) Write two equivalent expressions for the perimeter of each shape. Use brackets in one of your expressions.

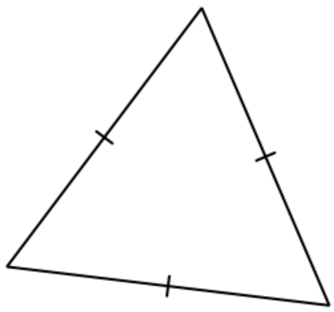




………………………… or …………………………   …………………………or…………………………

b) For each shape below, what is the side length such that the perimeter of each is $6(x + 2)$.





c) Draw two of your own examples of **regular** polygons with perimeters of $8(y - 1)$.

5. Barack walks a distance of $17(d+5)$ km then another $(d-2)$ km.
   Write a simplified expression for the total distance of his journey.

....

6. Expand and simplify the following expressions:

   a) $(x+2)+(x+4)$ $\equiv$ ....

   b) $(x+2)+(x-4)$ $\equiv$ ....

   c) $(x+2)-(x+4)$ $\equiv$ ....

   d) $(x+2)-(x-4)$ $\equiv$ ....

   e) $(x+2)-(4-x)$ $\equiv$ ....

   f) $(x-4)-(x+2)$ $\equiv$ ....

   g) $(x-4)+(x-2)-(x-2)$ $\equiv$ ....

   h) $2(x+2)-(x+4)$ $\equiv$ ....

   i) $4(x-2)-2(x-4)$ $\equiv$ ....

7. Expand and simplify the following expressions:

   a) $9(x - y) + 7(x + y)$ $\equiv$ ……………………………………………………

   b) $6(x + 2) + 3(2x - 4)$ $\equiv$ ……………………………………………………

   c) $16(2x + 3y) - 8(4x + 6)$ $\equiv$ ……………………………………………………

   d) $3(5 + x) - 8(2x - 4)$ $\equiv$ ……………………………………………………

   e) $4\left(\frac{x}{4} + 3\right) + 8\left(x - \frac{3}{4}\right)$ $\equiv$ ……………………………………………………

   f) $12\left(\frac{2}{3}x - \frac{5}{6}\right) - 9\left(\frac{x}{3} + 2\right) + 7$ $\equiv$ ……………………………………………………

8. Calculate the mean of the three expressions below:

   $6(x - 3)$ $11(7x - 9)$ $45 - 2x$

   …………………………

9. Two expressions have a sum of $13v + 19$.
   One of the expressions is $8(v - 2)$.
   What is the second expression?

   …………………………

10. Jeremy has incorrectly simplified the expression below. Correct his mistake.

$$
\begin{aligned}
&5(y+7)-2(y-2)\\
&\equiv 5y+35-2y-4\\
&\equiv 3y+31
\end{aligned}
$$

11. Expand and simplify the following expressions:

a) $3x(x+5)+4(x^2-3)$ $\equiv$ ....

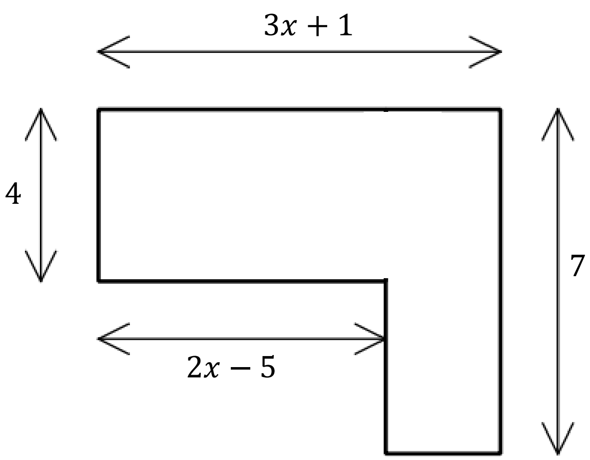
b) $4x(4x-11)+3x(6-2x)$ $\equiv$ ....

c) $3x(2+5x)-(15x-8x^2)+3$ $\equiv$ ....

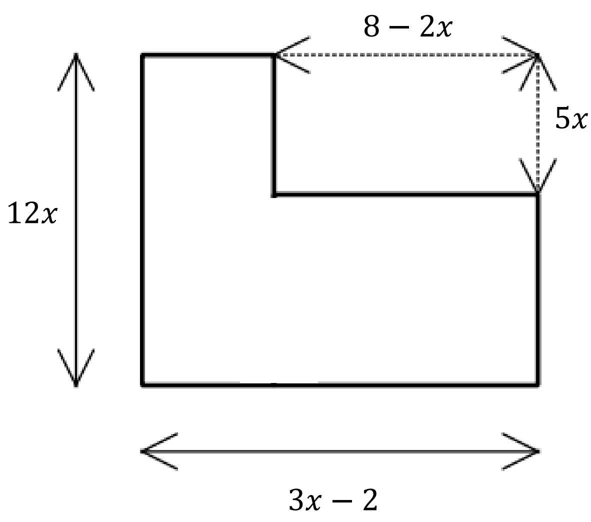
d) $8x\left(\frac{1}{4}x-5\right)-9x\left(\frac{2}{3}x-3\right)-4x$ $\equiv$ ....

12. Write simplified expressions for the area of each shape:

a)



b)



....

6.2 Factorising expressions with a single bracket

Concept Corner

inverse	factors

Complete the sentences below using the words in the box.

Factorising is the .... of expanding.

For example, the expression $12 - 2r$ can be written as the product of two ....

2 and $(6 - r)$ are the **factors**

$12 - 2r$ is the **product**

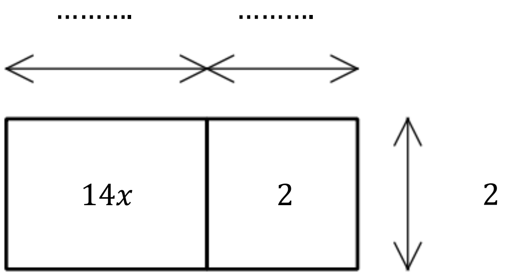
Expand

$$2(6 - r) \equiv 12 - 2r$$

Factorise

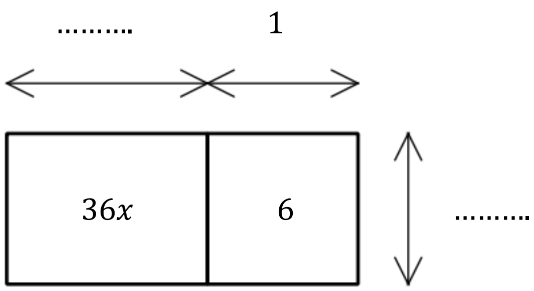
1. Complete the area models below with the missing dimensions and hence **fully factorise** each expression.

a)



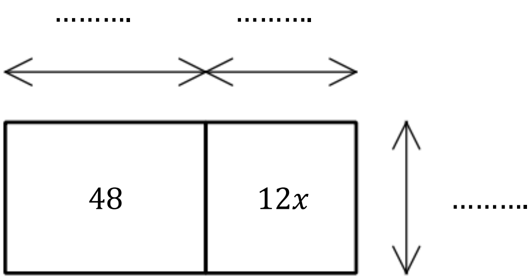
$14x + 2 \equiv 2(.......... + ...)$

b)



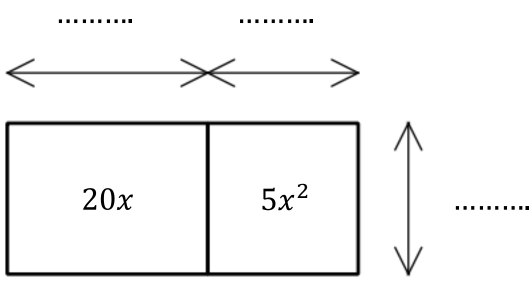
$36x + 6 \equiv$ ....

c)



$48 + 12x \equiv$ ....

d)



$20x + 5x^2 \equiv$ ....

2. Fully factorise the following expressions.

a) $15x + 20$

b) $32x - 16$

c) $42 - 14x$

d) $8x^2 + 4x$

3. Some students have made mistakes in their answers below. Correct each student's mistake.

Expand $5x(2x - 3)$

Answer: $7x^2 - 8x$

Correction: ……………………………………

Fully factorise $24x^2 - 8x$

Answer: $4x(6x - 2)$

Correction: ……………………………………

Fully factorise $14g^2 + 42g$

Answer: $14(g^2 + 3)$

Correction: ……………………………………

4. Fully factorise the following expressions:

a) $10t + 2t^2$ $\equiv$ ………………………………………………………

b) $9s^2 + s$ $\equiv$ ………………………………………………………

c) $13u - 52u^3$ $\equiv$ ………………………………………………………

d) $4x^3 + 12x - 6$ $\equiv$ ....

e) $2ab + ad$ $\equiv$ ....

f) $14qr^2 - 21q^2r$ $\equiv$ ....

g) $4xy - 12xy^2 + 28x^2$ $\equiv$ ....

h) $7(3x + 5) + 2(3x + 5)$ $\equiv$ ....

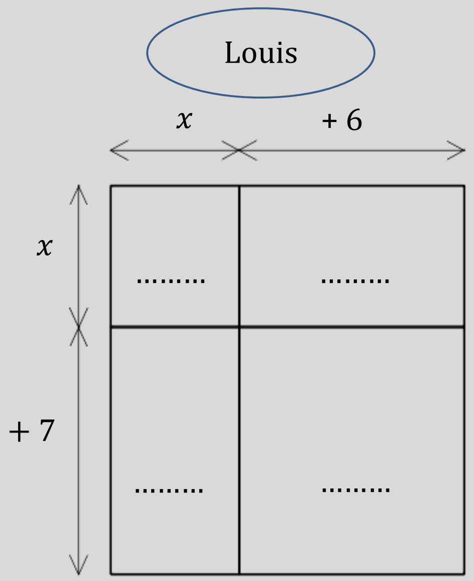
i) $4p(2y - 7) + 5(2y - 7)$ $\equiv$ ....

6.3 Expanding products of binomials

Concept Corner

Louis and Luke have used two different approaches to expand $(x + 6)(x + 7)$.

Fill in the gaps



Luke

$(x + 6)(x + 7)$

$\equiv x(x + 7) + 6(x + 7)$

$\equiv x^2 +$......... + ...+ ....

$\equiv x^2 +$ .... $+ 42$

Which method did you prefer? Can you do it a different way?

1. Write the following expressions into the correct column in the table below:

$4x^2 - 9$ $\quad$ $(x+1)(x-1)$ $\quad$ $a^3 + 9a$

$2x^2 - 6x$ $\quad$ $8y$

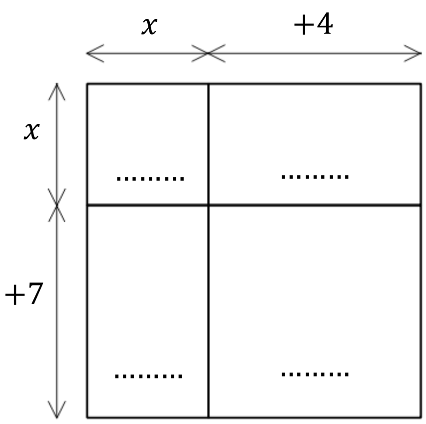
$3x + 3$ $\quad$ $c^2 + 11c + 28$

$6x^2 + 17x + 12$

Linear	**Quadratic**	**Neither**

2. Expand the following

$(x+4)(x+7) \equiv$ ……………………………….



$(x+4)(x+7)$

$\equiv x(x+7) + 4(x+7)$

……………………………………………………

……………………………………………………

3. Expand and simplify

   a) $(x + 5)(x + 6)$

   b) $(x + 6)(x + 5)$

   Are your answers the same or different? Explain why.

4. Expand and simplify the following expressions:

   a) $(x + 4)(x + 3)$

   b) $(x + 4)(x + 2)$

   c) $(x + 4)(x + 1)$

   d) $(x + 4)x$

   e) $(x + 4)(x - 1)$

   f) $(x + 4)(x - 2)$

   g) $(x + 4)(x - 3)$

   h) $(x + 4)(x - 4)$

5. Expand and simplify the following expressions:

   a) $(x+9)(x-2)$

   b) $(y+3)(y-6)$

   c) $(p+5)^2$

   d) $(4+x)(x-5)$

   e) $(c-8)(c-7)$

   f) $(4-h)(h+3)$

   g) $(x+2)(x+3)(x+4)$

   h) $(x+y-2)(x-y+5)$

6. Expand and simplify the following expressions:

   a) $(2x+5)(x+3)$

   c) $(4x+5)(x+3)$

   b) $(3x+5)(x+3)$

   d) $(4x+5)(x-3)$

e) $(4x+5)(3-x)$

h) $(5-3x)(5+3x)$

f) $(2x+3)(3x-5)$

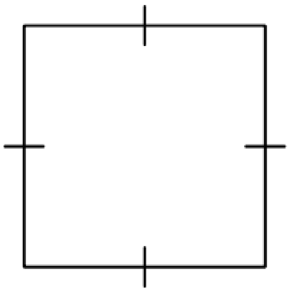
i) $(x+4)(x-4)(2x+1)$

g) $(3x-5)^2$

j) $(x^2+4x-5)(x^2+4x+5)$

7. Write a simplified expression for the area of each shape.

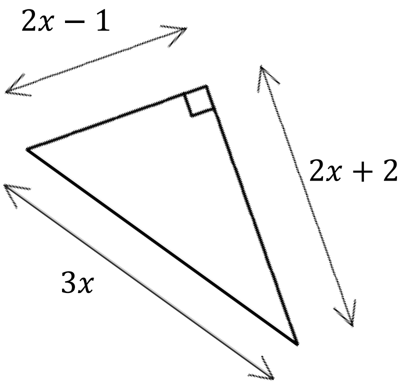
a)



$2x+9$

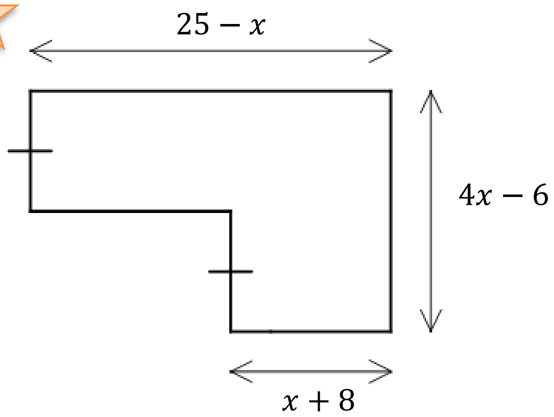
……………………………………………

b)



……………………………………………………

c)



……………………………………………………

Reflections

This space is for you to write your reflections on the whole unit on expanding and factorising

You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future

Chapter 7: Changing the subject of a formula

7.1 Substitution

1. Put the following expressions in order of size, starting with the smallest:

   a) When $\boldsymbol{a} = \mathbf{1}$.

$3 + a$	$3a$	a^2	$\frac{a}{3}$	$3 - a$

   b) When $\boldsymbol{a} = \mathbf{-1}$.

$3 + a$	$3a$	a^2	$\frac{a}{3}$	$3 - a$

2. Complete the table by substituting the given values into each expression.

$\boldsymbol{x}$	$\boldsymbol{6x}$	$\boldsymbol{2x - 3}$	$\boldsymbol{6x - 3}$	$\boldsymbol{3(2x - 3)}$
5				
	-30			
-0.5				

3. Substitute the given values into each expression. Some have been completed for you.

$\boldsymbol{y}$	$\boldsymbol{3y - 2}$	$\boldsymbol{\frac{y + 3}{4}}$	$\boldsymbol{3(1 + 6y)}$	$\boldsymbol{3y^2}$	$\boldsymbol{\frac{12}{y^2 - 1}}$
5					
	-11			27	
$\frac{3}{2}$		$\frac{9}{8}$			

7.2 Forming expressions, equations and formulae

1. Here are the rules for completing the algebra grid.

This is the **sum** of the middle row →

This is the **product** of the middle row →

$3x+3$	
$2x$	$x+3$
$2x^2+6x$	

Use these rules to complete the grids below:

a)

2	$x+2$	x	$x+2$	x	$2x+2$

b)

$5x+8$		$6x+3$		$10x+3$	
$5x+3$		$5x+3$		$5x+3$	

c)

$3x-2$		$3x-2$		$3x-2$	
$12x-8$		$-12x+8$		$-12x^2+8x$	

d)

x	$x+5$	$x+1$	$x+5$	$x-1$	$x+5$

e)

$2x+6$		$3x+6$		$4x+6$	
$x+5$		$2x+5$		$2x+5$	

f)

$x-7$		$x-7$		$x-7$	
x^2-7x		$x^2-14x+49$		x^2-49	

g)

$-2x^2+4x+5$		$2x^2$		$2x^3-5x-4$	
		x^2+6x			
$20x-8x^3$				$20x-8x^3$	

Concept Corner - Use the words in the box to help you fill in the gaps.

variables
equal
expressions
equals

An **expression** is a collection of terms without an .... sign.

For example: $3x + 4$

An **equation** shows that two or more .... are ....
Equations always have an equals sign.

For example: $11 = 3x + 4$

A **formula** expresses a relationship between two or more ....

For example: $y = mx + c$

2. Write your answers in their simplest form.

a)

$\times$	$5x$	$\frac{x}{4}$	$x + 2$
12			$12x + 24$
-8			
$\frac{1}{2}$			

b)

$\times$	$x - 3$	x^2	$\frac{4}{x}$
12	$12x - 36$		
$\frac{1}{2}$		$\frac{x^2}{2}$	
x			

c)

$\times$	$5x$	$-\frac{x}{6}$	$x - 6$
-0.5			
101			$101x - 606$
y	$5xy$		

d)

$\times$	x	$x + 1$	$x - 1$
x			
		$xy + y$	
	x^2y		

e)

$\times$	$4(c+2)$	$5c$		
		$15c$	$3c+9$	$3c^2 - 15c$
	$8c^2 + 16c$		$2c^2 + 6c$	$2c^3 - 10c^2$
$c+5$			$c^2 + 8c + 15$	
		$15c - 5c^2$	$9 - c^2$	

3. Decide if each statement below is **true** or **false**:

a) $p + 7 \equiv 7p$

b) $11g \equiv 3g + 8g$

c) $6x + 3y = 9xy$

d) $6x \times 3y \equiv 18xy$

e) $2b \times b \equiv 3b$

f) $5(r-2) \equiv 5r - 2$

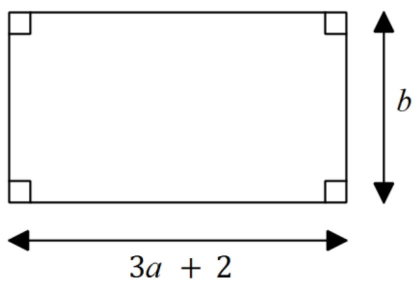
g) $7h(h-3) \equiv 7h^2 - 21$

h) $-8a(a+2) \equiv -8a^2 - 16a$

i) $2(w+3) + 3(w-2) \equiv 5w + 1$

j) $6(y-2) - (y+2) \equiv 5y - 14$

4. Write down expressions for the perimeter and area of the rectangle below.



5. Given that $17 \times 23 = 391$, decide if the following statements are **true** or **false**:

a) $23 \times 17 = 391$

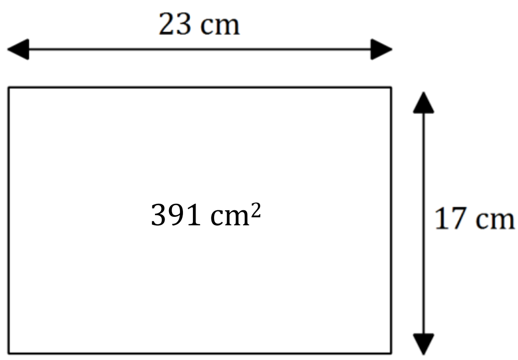
b) $\frac{391}{17} = 23$

c) $\frac{23}{391} = 17$

d) $170 \times 23 = 3910$

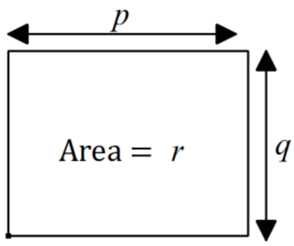
e) $17 = 391 - 23$

f) $17 \times 23 + 100 = 491$

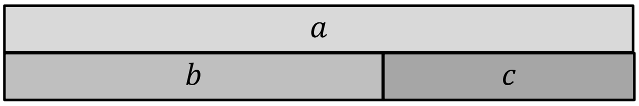


6. The lengths of the rectangle are represented by letters p and q. The area is represented by the letter r.

Write **three** statements to describe the relationship between p, q and r.



7. The bar model below shows the relationship between a, b and c.



a) Circle the equations that are represented by this bar model.

$\boxed{a = b + c}$ $\boxed{a = b - c}$ $\boxed{b = a - c}$ $\boxed{c = b - a}$

$\boxed{a \div c = b}$ $\boxed{a - c = b}$ $\boxed{c - b = a}$ $\boxed{c = a - b}$

$\boxed{a - b = c}$ $\boxed{cb = a}$ $\boxed{b + c = a}$ $\boxed{c - a = b}$

8. Given that $\boldsymbol{x \equiv cp}$, decide if the following statements are **true** or **false**:

a) $\frac{x}{p} \equiv c$

b) $\frac{c}{x} \equiv p$

c) $x - c \equiv p$

d) $p \equiv cx$

e) $cx \equiv c^2p$

f) $\frac{c}{p} \equiv \frac{x}{p^2}$

9. Write down a **formula** for each rule below:

a) y is 5 more than x

b) D is 7 times T

c) P is the sum of $2l$ and $2w$

d) y is 3 less than four times x

10. Classify each of the following statements by ticking the appropriate column.

	Expression	**Equation**	**Formula**
$3x - 5$			
$7x - 3 = 20$			
$y = mx + c$			
$C = \pi d$			
$y = x$			
$x^2 - 5x + 4$			
$A = \pi r^2$			
$2p^2 - 4p + 5 = 0$			
$6x^2y - 3y^2x$			

7.3 Working with equations

1. Solve the following equations.

a) $4c = 17$

b) $3p + 7 = 34$

c) $26 = 5t - 8$

d) $\frac{y}{8} = 0.6$

e) $20 - 6x = 5$

f) $\frac{10}{g} = 8$

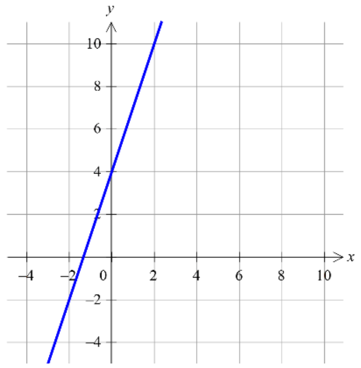
g) $12 = 3(2p + 5)$

h) $2(4y - 3) = -12$

i) $x^2 = 36$

j) $x^2 + 20 = 45$

2. The equation $y = 3x + 4$ represents a straight line.



a) Find the value of y when $x = 2\frac{1}{3}$.

b) Find the value of x when $y = 20$.

c) Write down the gradient and y-intercept of the line.

3. Temperatures can be converted from degrees Fahrenheit (F) to degrees Celsius (C) using the formula $F = \frac{9}{5}C + 32$.

   a) Convert $32°C$ to degrees Fahrenheit.

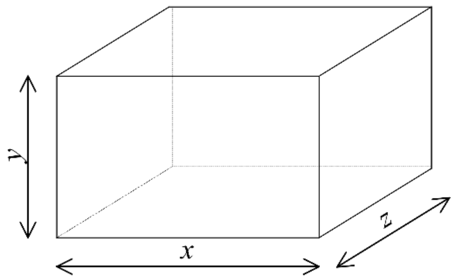
   b) Convert $-10°F$ to degrees Celsius.

4. The formula $A = \pi r^2$ is used to calculate the area of a circle.

   a) Calculate the area when the $r = 8$ cm.
   Give your answer accurate to 1 decimal place.

   b) Calculate the radius when the area is 20 cm^2.
   Give your answer accurate to 1 decimal place.

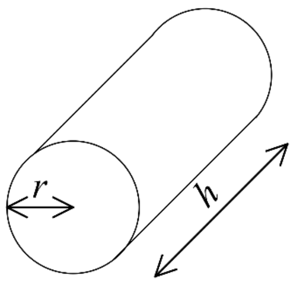
5. The formula $A = 2xy + 2xz + 2yz$ can be used to calculate the total surface area of cuboid with edges of lengths x, y, and z.
   a) Calculate the total surface area of a cuboid with edges of length 8 cm, 4 cm and 12 cm by **substituting into the formula** above.
   Give your answer in cm^2.



   b) Convert your answer to part a) into m^2.

   c) A cuboid has a total surface area of 494 cm^2.
   Give that it has edges $x = 5$ cm, $y = 12$ cm, calculate the length of the remaining edge, z cm.

6. The formula $A = 2\pi r^2 + 2\pi rh$ is used to calculate the surface area of a cylinder.
   a) Calculate the surface area when $r = 6$ cm and $h = 10$ cm.
   Give your answer accurate to 3 significant figures.



   b) Calculate the value of the height (h) when the surface area is 300 cm^2 and $r = 5$ cm.
   Give your answer accurate to 3 significant figures.

7.4 Rearranging formulae

1. Identify the 'subject' in each of the following equations:

   a) $A = \pi r^2$ Subject: ……………………………

   b) $y = mx + c$ Subject: ……………………………

   c) $\sqrt{\frac{A}{6}} = x$ Subject: ……………………………

   d) $S = \frac{D}{T}$ Subject: ……………………………

   e) $\pi d = C$ Subject: ……………………………

2. Make $\boldsymbol{x}$ the subject of each formula given below:

   a) $y = x - 9$

   b) $y = x + 9$

   c) $x + 8 = y$

   d) $x - 3 = y$

   e) $y = \frac{x}{7}$

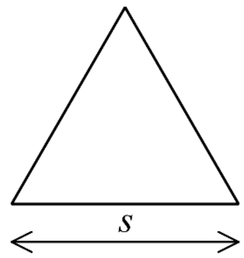
   f) $y = 5x$

   g) $\frac{x}{9} = y$

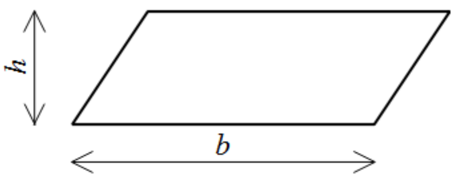
   h) $y = 5 - x$

3. Rearrange the formulae below as required.

   a) $P = 3s$
   Make s the subject.



   b) $A = bh$
   Make h the subject.



4. Make $\boldsymbol{x}$ the subject of each formula given below:

a) $y = 2x - 6$

b) $2x - 6 = y$

c) $y = 6x - 2$

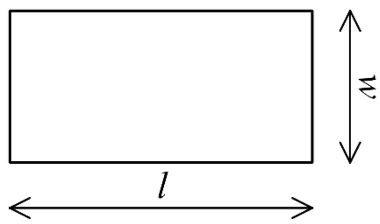
d) $y = \frac{x}{6} + 2$

e) $y = \frac{x}{6} - 2$

f) $\frac{x}{2} - 6 = y$

5. Rearrange the formulae below as required.

a) $P = 2w + 2l$
Make w the subject.



b) $S = \frac{D}{T}$
i. Make D the subject.

ii. Make T the subject.

c) $A = \frac{bh}{2}$
Make h the subject.

d) $V = x^3$
Make x the subject.

e) $y = mx + c$

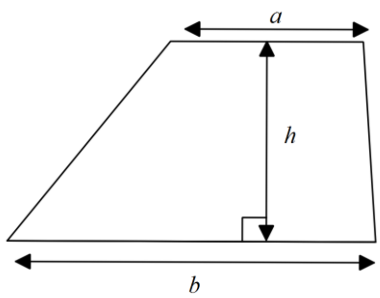
i. Make c the subject.

ii. Make x the subject.

f) $A = \pi r^2$
Make r the subject.

6. Rearrange the formula for the area of a trapezium.

$$A = \frac{h}{2}(a + b)$$



a) Make h the subject.

b) Make a the subject

7. Circle the formulae that are equivalent to $\boldsymbol{A = 2\pi r^2 + \pi dl}$.

$$\frac{A - \pi d}{2\pi r^2} = l$$

$$A - 2\pi r^2 - \pi l = d$$

$$\frac{A - 2\pi r^2}{\pi d} = l$$

$$\sqrt{\frac{A - \pi dl}{2\pi}} = r$$

$$\sqrt{\frac{A}{\pi dl} 2\pi} = r$$

$$A = \pi(2r^2 + dl)$$

8. Each student below has tried to make ***y* the subject** of a formula.
Highlight and then correct their mistakes.

$$x = 8yz$$
$$x \div z = 8y$$
$$x \div z \div 8 = y$$

Correction:

$$x = \frac{2y - 20}{5}$$
$$x + 20 = \frac{2y}{5}$$
$$5(x + 20) = 2y$$
$$\frac{5(x + 20)}{2} = y$$

Correction:

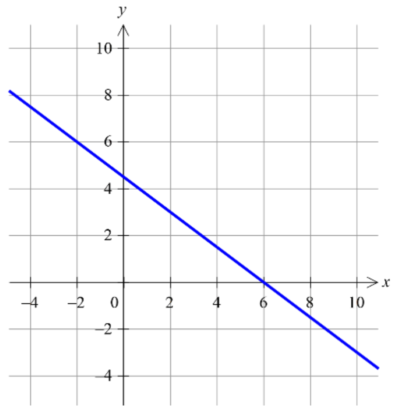
$$V = 6y^2$$
$$\sqrt{V} = 6y$$
$$\frac{\sqrt{V}}{6} = y$$

Correction:

11. Circle the odd one out.

a)	$S = \frac{D}{T}$	$S = \frac{T}{D}$	$ST = D$	$T = \frac{D}{S}$
b)	$\frac{C}{\pi} = d$	$\pi d = C$	$\pi = \frac{C}{d}$	$\frac{\pi}{C} = d$
c)	$\frac{y + c}{m} = x$	$y = mx + c$	$y - c = mx$	$x = \frac{y - c}{m}$

7.5 Straight line graphs – changing the subject



1. The equation $3x + 4y = 18$ represents a straight line.
   a) Find the value of y when $x = 5$.

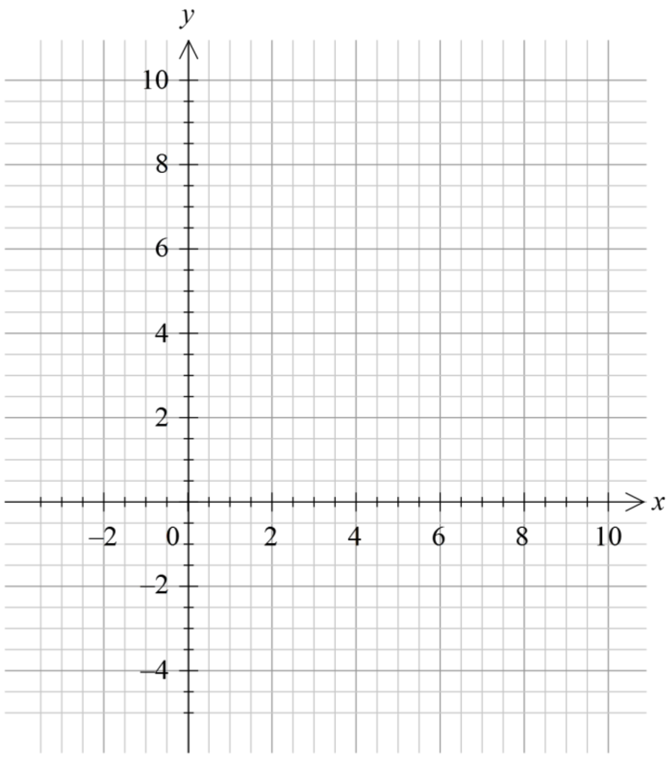
   b) Find the value of x when $y = 6$.

   c) Find the gradient and y-intercept of the line

2. Plot the graphs of the following three equations on the axes below.
   a) $y = 2x - 1$ b) $3x + 4y = 12$ c) $2y - 3x + 2 = 0$

   Hence or otherwise, write down the gradient of each line.

   a) …………………………….. b) …………………………….. c) ……………………………..



3. Rearrange each of the following line equations into the form $y = mx + c$.
By comparing the gradient and y-intercept, find the odd one out in each set of three.

a)

$2y = 4x + 8$ $\qquad$ $y + 2x = 11$ $\qquad$ $y - 2x = 9$

……

……

……

b)

$3y = 5x + 2$ $\qquad$ $10x - 6y = 11$ $\qquad$ $3y + 5x = 2$

……

……

……

c)

$3y + 12x = 14$ $\qquad$ $y = 7 + 4x$ $\qquad$ $2y + 3x = 14$

……

……

……

4. Write down an equation of a line parallel to each of the following lines, leaving your answer in the form $y = mx + c$:

a) $y = 4x - 8$

b) $x + 4y = 8$

c) $4(x + y) = 8$

d) $x - 8 + \frac{y}{4} = 0$

5. Write down an equation of a line with the same y-intercept as each of the following lines leaving your answer in the form $y = mx + c$:

a) $y = 12 - 3x$

b) $x = 12 - 3y$

c) $x = \frac{12-y}{3}$

Reflections

This space is for you to write your reflections on the whole unit on changing the subject of a formula.

You may wish to write about:

- Things you've learnt
- Things you found difficult
- Other areas of maths you used in this topic
- Topics you need to revisit/revise in the future